By: Dr. Daniel A. Wendichansky Bard

Centro de Transferencia de Tecnología en Transportación

ABC: Vigas Pretensadas para Puentes
Principios Básicos de Diseño

Pre-Stressed Concrete Structures
Basic Concepts

PRE-STRESSED CONCRETE SEMINAR

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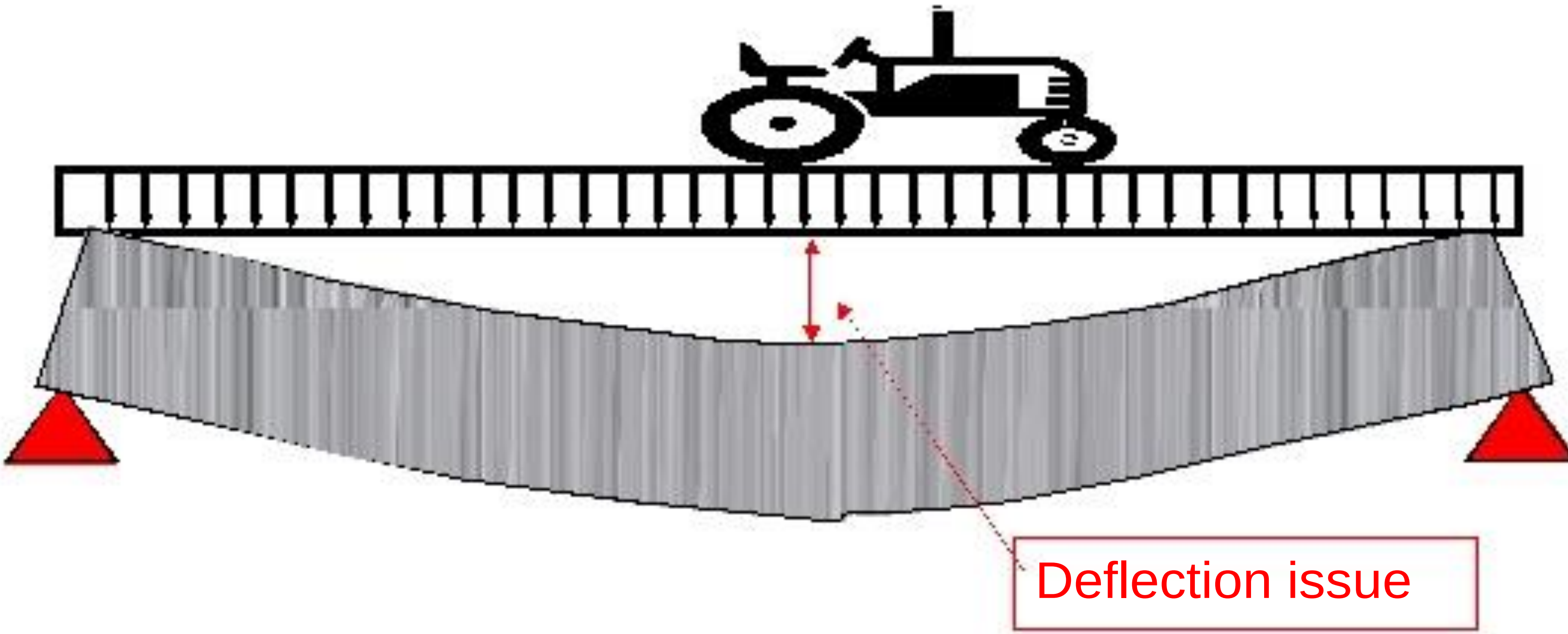
Basic Principles of Reinforced Concrete Design



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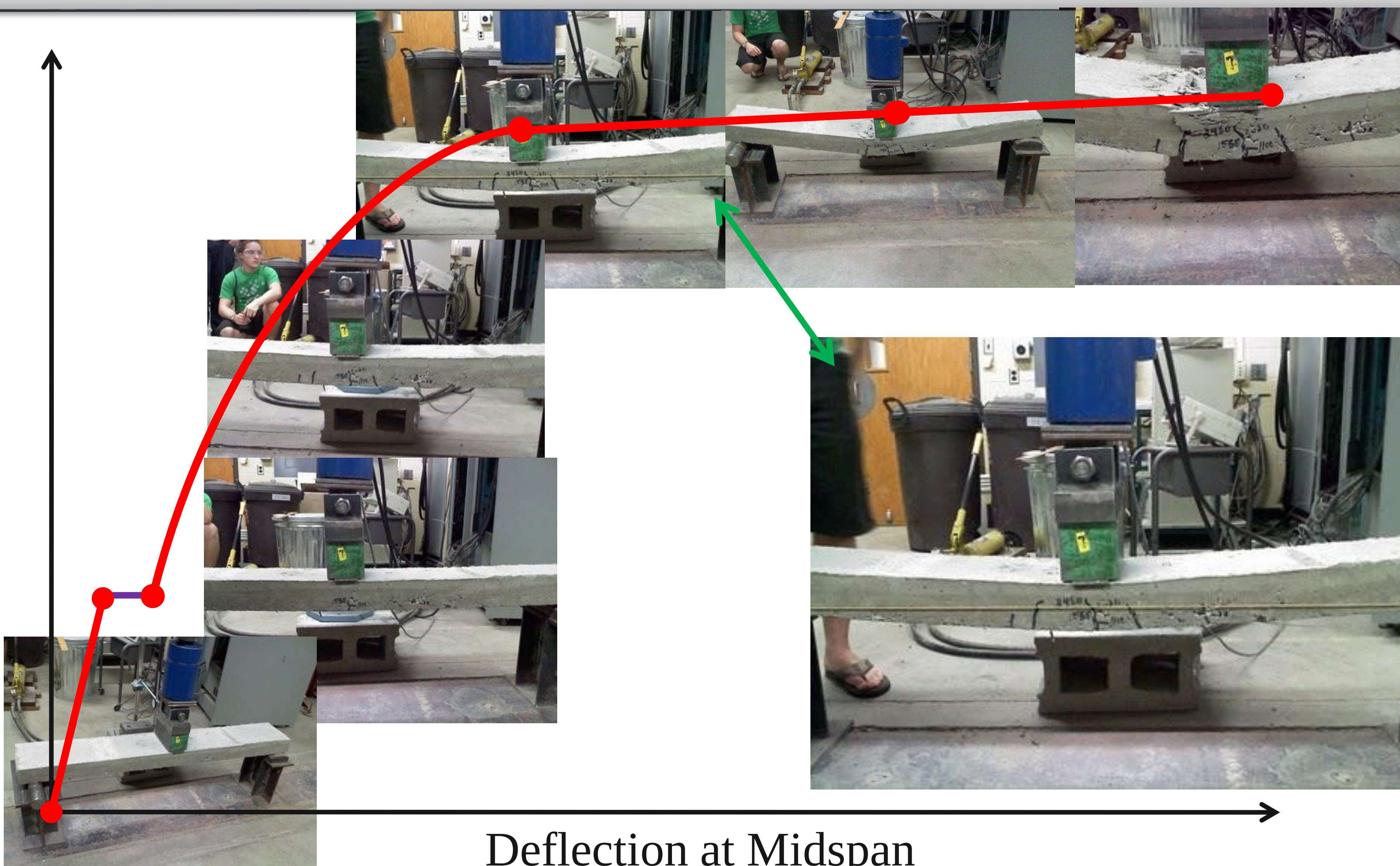
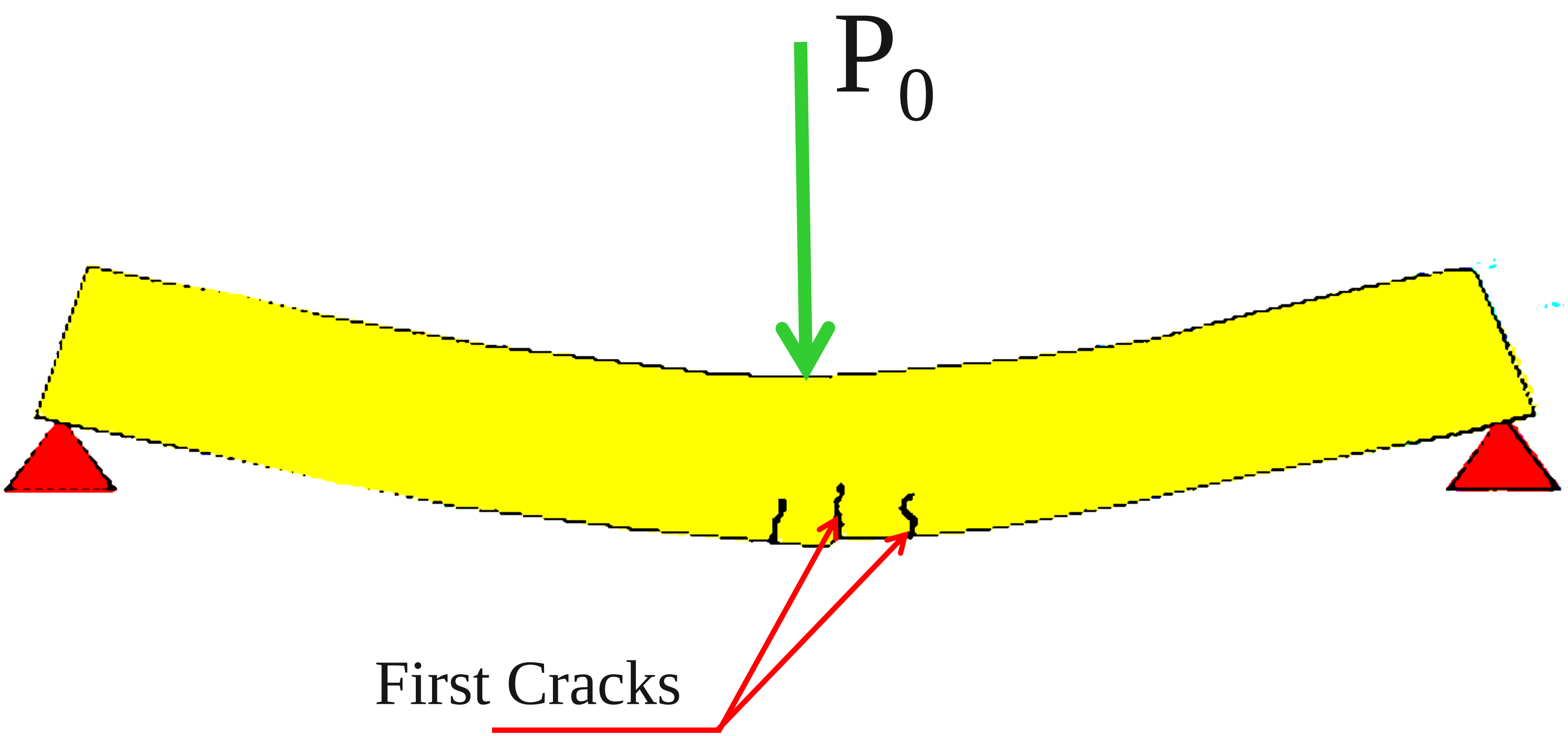
Deflection Response of RC Beams



Concrete Beam Behavior – Initial Stage

First Cracks Appear, End of Elastic beam behavior. The principles learned during the Mechanical of Material Courses are not longer useful.



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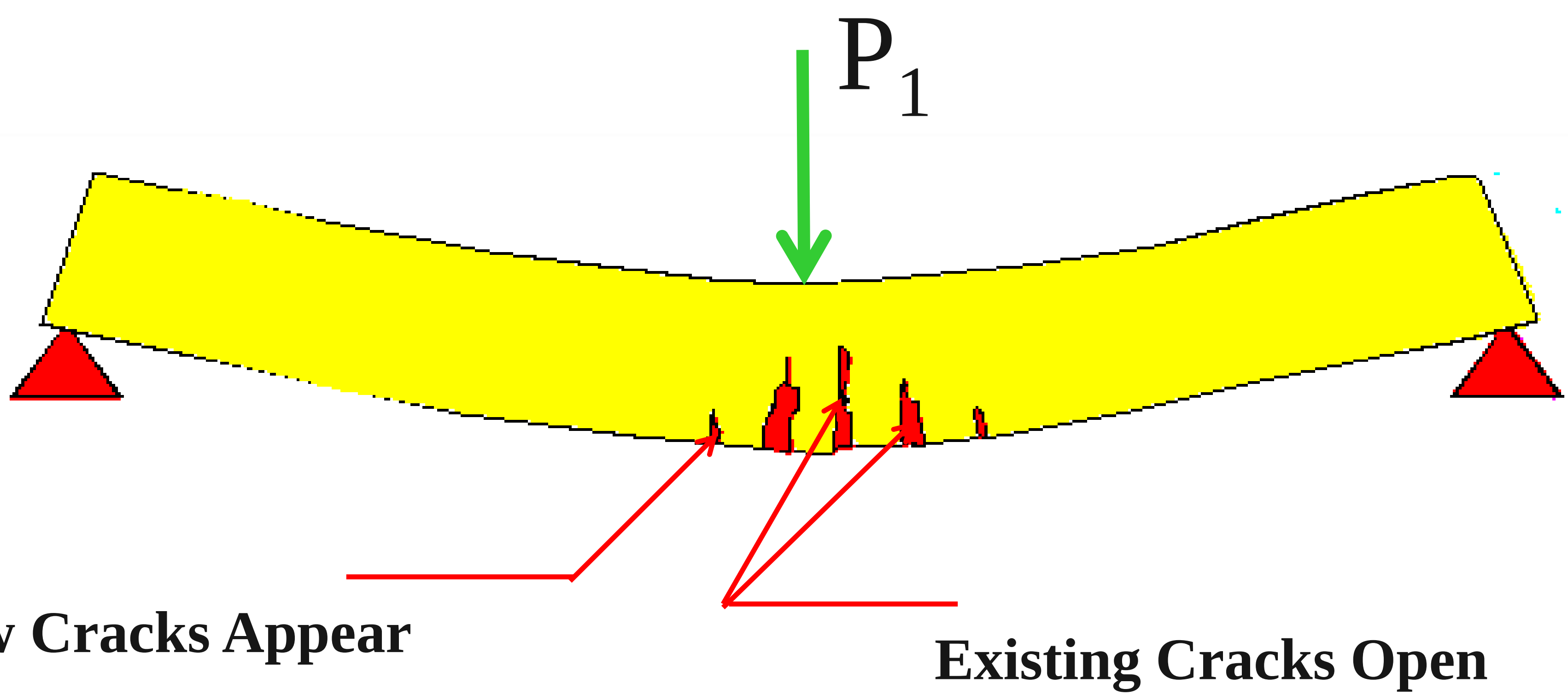
Concrete Beam Behavior – Intermediate Stage

New Cracks Appear. The first cracks open more and more each time that the load increase . There is not sign of failure of the concrete located in the compression side of the beam (Top Side of the beam for this load condition)



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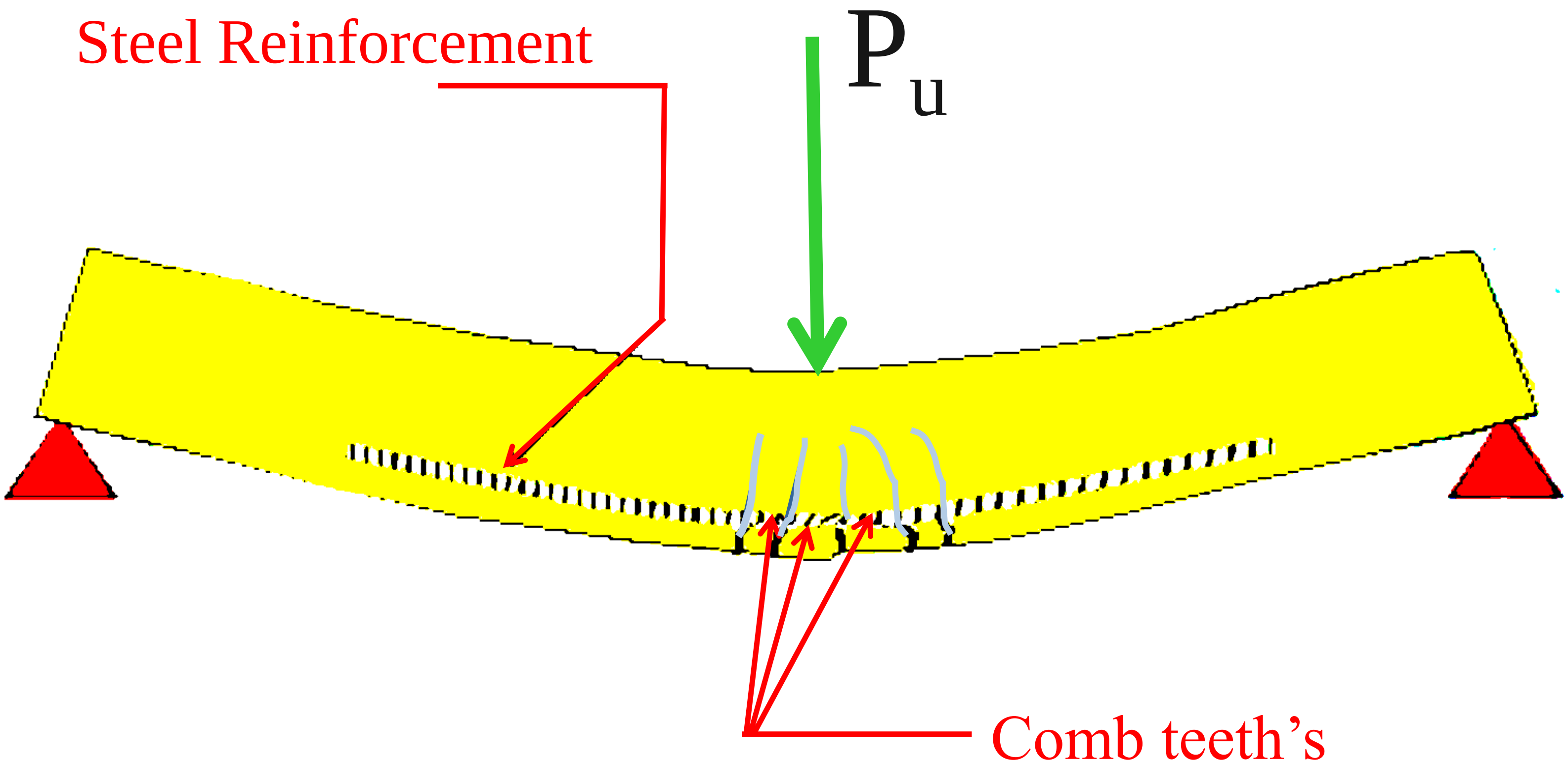
Concrete Beam Behavior – Ultimate Stage



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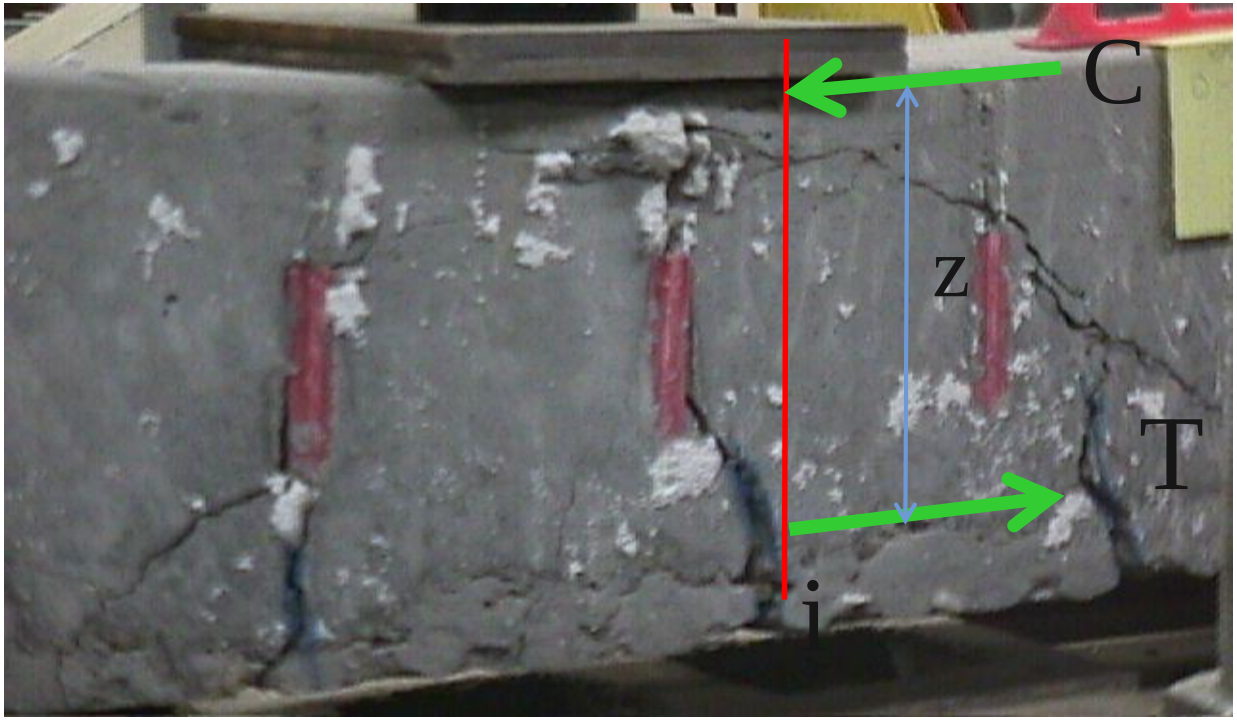
Concrete Beam Behavior – Ultimate Stage

At ultimate load (P_u) The resisting moment can't be computed using the principles of mechanics of materials due that the section has suffer a considerable inelastic deformation. For this case the resisting moment shall be computed using equilibrium principles. Or in other words the resisting moment can easily computed at any section " i " as the value of "C" or "T" multiplied by "z"



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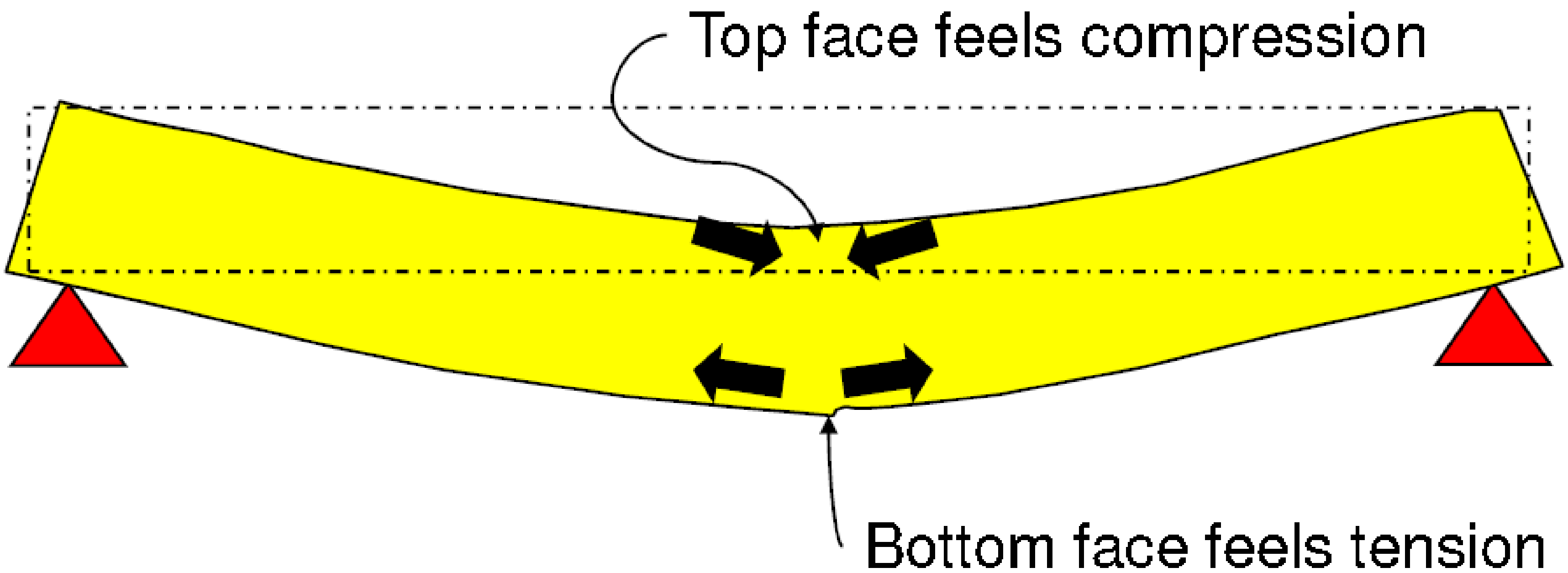
Stresses Induced by the Acting Load



EFFECT OF THE LOADS

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Minimum Thickness

ACI - Table 9.5(a) – Minimum thickness, h				
Member	**Simply supported**	**One end continuous**	**Both ends continuous**	**Cantilever**
Beams or ribbed one-way slabs	**ℓ/16**	**ℓ/18.5**	**ℓ/21**	**ℓ/8**

h_{min} = 50'/16 = 3.125 ft = **37.5" > 28"** ; L / h_{min} = **50'/ 3.125'** = 16

It is curious, I can obtaining the capacity needed with a 12"x 28" beam but probably if I use this one, the deformation will be greater than the code limits. If I increase the beam size the opposite situation occurs. I am wondering if something different could be done???

Result Discussions

$w_{DL} = b*h*\gamma = (12''*38'')/144*.15\ k/ft^3 = 0.475\ k/ft$ $\quad W_{Sup\text{-}Imp} = 0.560\ k/ft$

$M_u = \{(1.2*(w_{DL} + w_{Sup\text{-}Imp}) + 1.6w_{LL}) * L^2\}/8$

$w_{LL} = (8*M_u/L^2 - 1.2*(w_{DL} + w_{Sup\text{-}Imp})/1.6$

$W_{LL\text{-}bal\text{-}38''} = 0.52\ k/ft$ $\quad$ Vs. $W_{LL\text{-}bal\text{-}28''} = 0.61\ k/ft$

$W_{LL\text{-}tens\text{-}38''} = 0.49\ k/ft$ $\quad$ Vs. $W_{LL\text{-}tens\text{-}28''} = 0.59\ k/ft$

It is curious, I can obtaining the capacity needed with a 12"x 28" beam but probably if I use this one, the deformation will be greater than the code limits. If I increase the beam size the opposite situation occurs. I am wondering if something different could be done???

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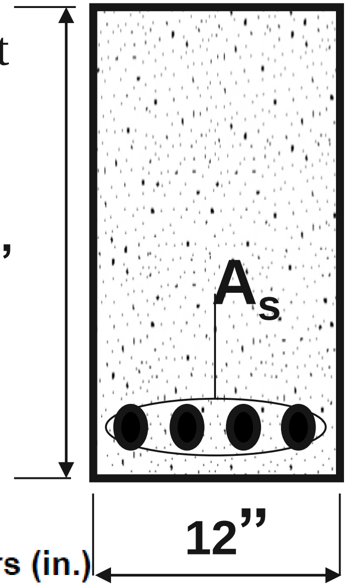
SAFE SUPERIMPOSED SERVICE LOAD

$W_{sup\text{-}Imp} + W_{LL\text{-}bal\text{-}38"}$=0.56 k/ft + 0.52 k/ft =1.08 k/ft

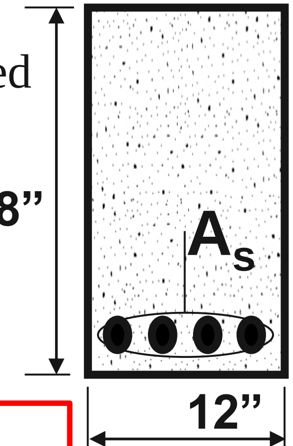
Reinforced Concrete

38”



Pre-stressed Concrete

28”



Key

3553 – Safe superimposed service load, plf.
0.4 – Estimated camber at erection, in.
0.2 – Estimated long-time camber, in.

Table of safe superimposed service load (plf) and cambers (in.)

Designation	No. Strand	y_s(end) in. y_s(center) in.	Span, ft																	
			16	18	20	22	24	26	28	30	32	34	36	40	42	44	46	48	50	52
12RB16	58-S	3.00 3.00	3553 0.4 0.2	2772 0.5 0.2	2212 0.6 0.2	1799 0.8 0.2	1484 0.9 0.3	1239 1.0 0.3	1045 1.1 0.3											
12RB20	88-S	3.00 3.00	6163 0.4 0.2	4825 0.5 0.2	3867 0.6 0.3	3159 0.7 0.3	2620 0.9 0.4	2201 1.0 0.4	1868 1.1 0.4	1600 1.3 0.5	1380 1.4 0.5	1198 1.5 0.5	1046 1.7 0.5							
12RB24	108-S	3.60 3.60	8950 0.4 0.2	7018 0.4 0.2	5636 0.5 0.3	4613 0.7 0.3	3835 0.8 0.3	3230 0.9 0.4	2749 1.0 0.4	2362 1.1 0.5	2045 1.3 0.5	1782 1.4 0.6	1562 1.5 0.6	1375 1.6 0.6	1216 1.8 0.6	1079 1.9 0.7	960 2.0 0.6			
12RB28	128-S	4.00 4.00		9781 0.4 0.2	7866 0.5 0.2	6448 0.6 0.3	5370 0.7 0.3	4532 0.8 0.4	3866 0.9 0.4	3329 1.0 0.5	2890 1.2 0.5	2525 1.3 0.6	2220 1.4 0.6	1962 1.5 0.7	1741 1.7 0.7	1552 1.8 0.7	1387 1.9 0.8	1244 2.0 0.8	1118 2.1 0.8	1006 2.2 0.8

PRE-STRESSED CONCRETE SEMINAR

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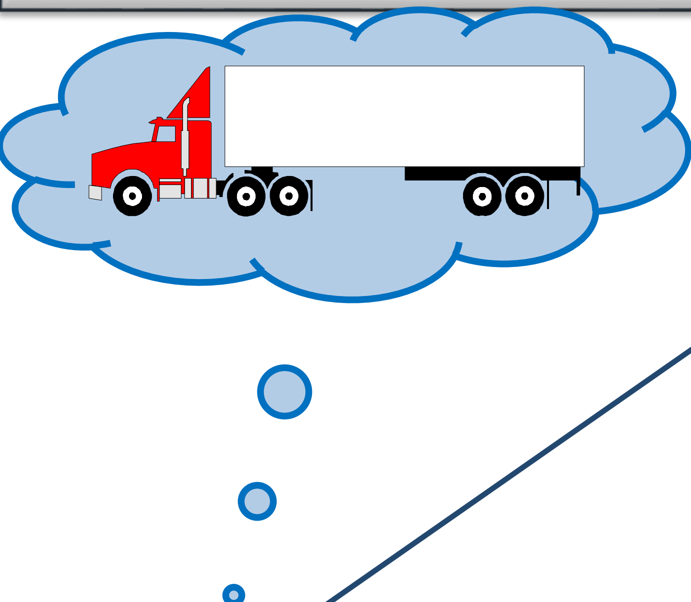
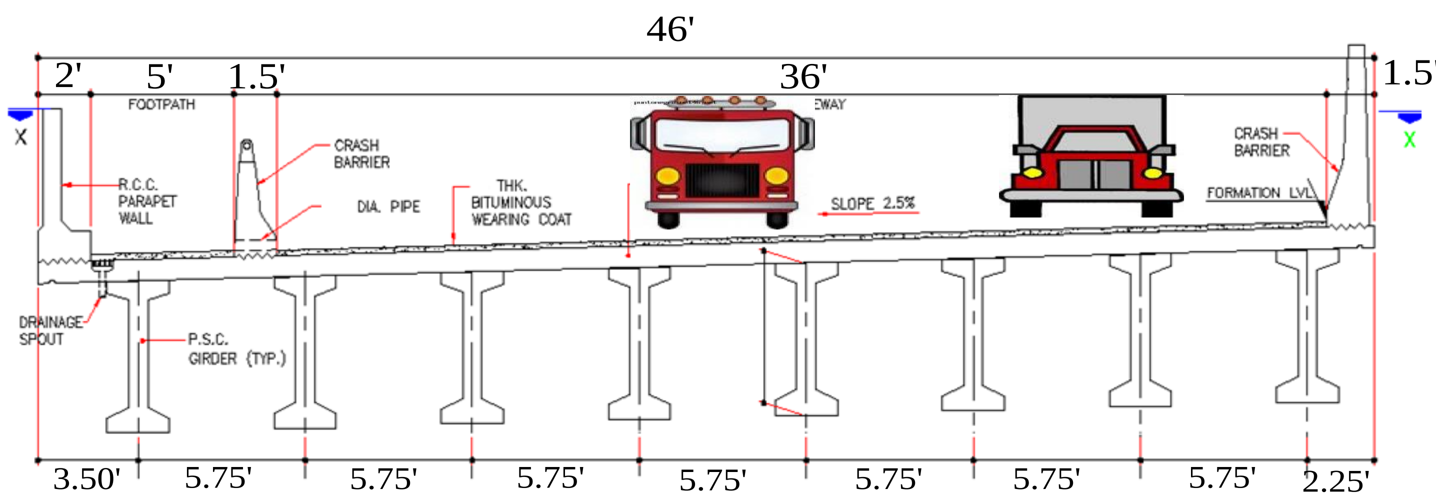
Result Discussions



Clearly here we have a problem. If I want to maximize the capacity of the structure to sustain very heavy Live Loads Like in bridges, I need to reduce the effect of the dead load. But If I reduce the size of the elements the deflections will increase and I will obtain deflections values that will be greater than the values prescribed by the code. Let me see if pre-stressing the structure I solve the problem.

Dr. Wendy



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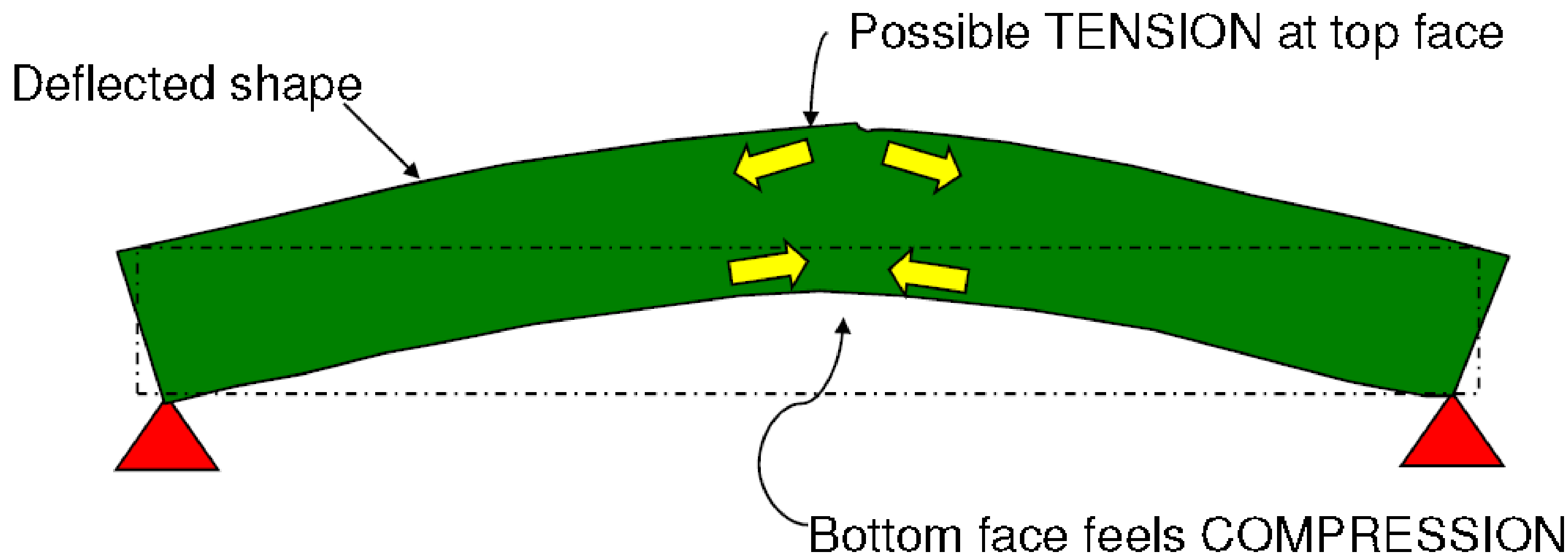
Pre-Stressed Effects



EFFECT OF THE POST-TENSIONING

OPPOSITE EFFECT FROM THE LOADS

Cracks, deflections, loadings, behavior is controlled by active reinforcement called post-tensioning.

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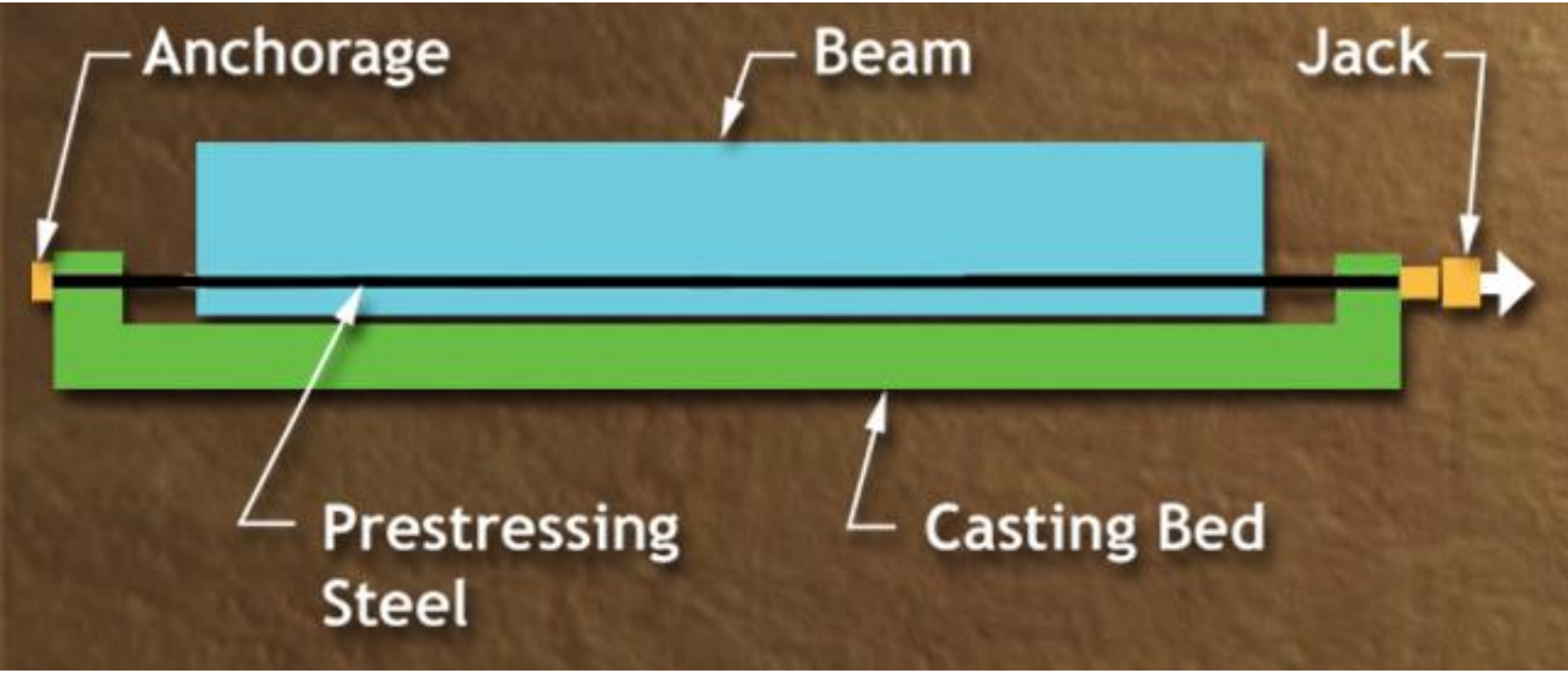
Pre-Stressed Load Application Procedure



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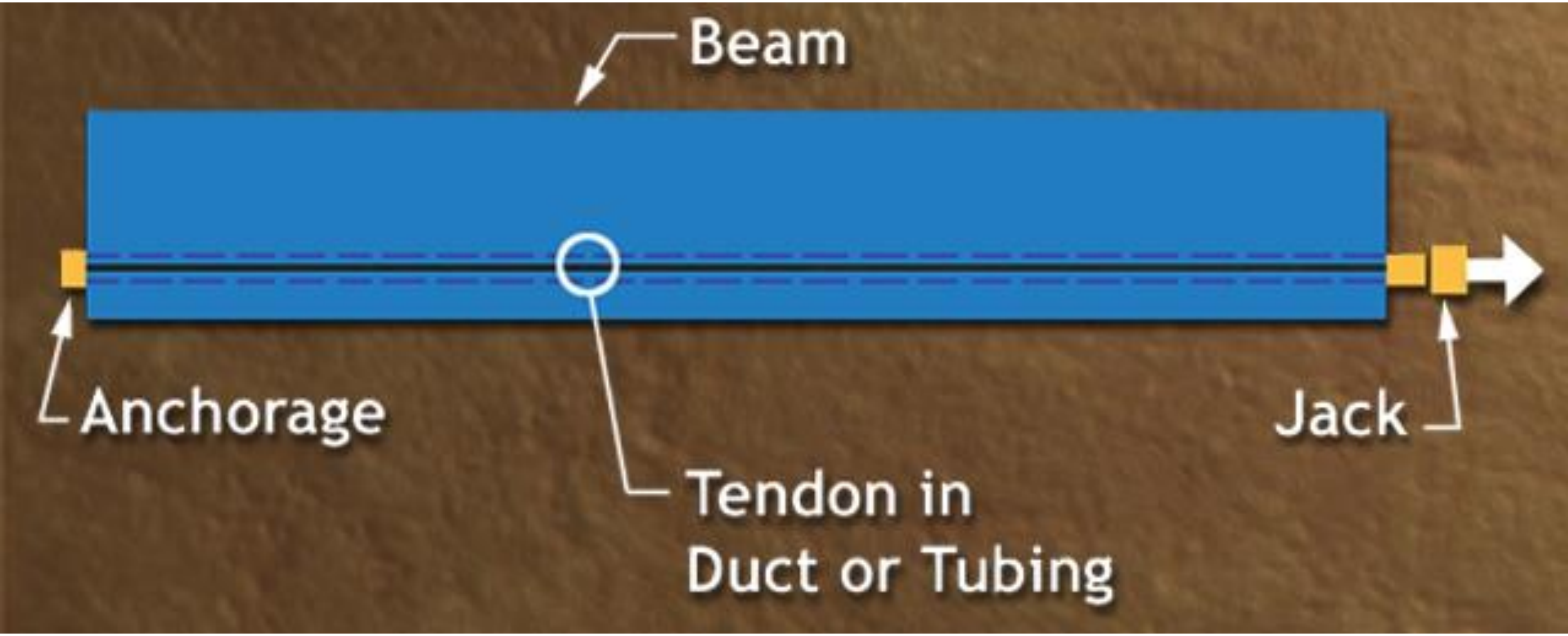
Post-tensioned Load Application Procedure



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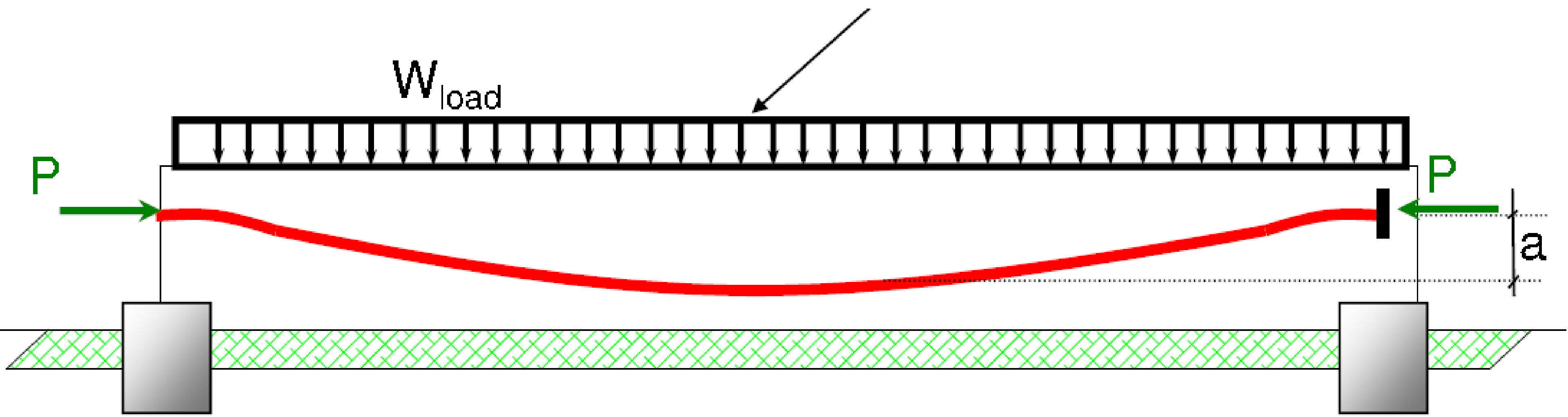
Equivalent Pre-Stressed Effects



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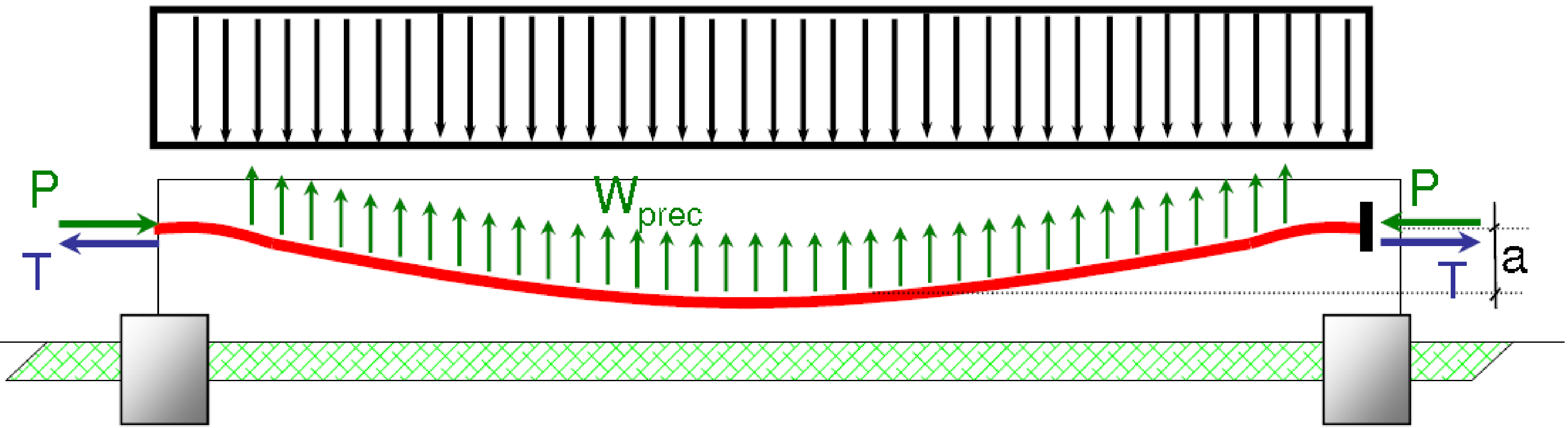
Equivalent Pre-Stressed Effects



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Pre-Stressing, Post-Tensioning Advantage

Main Advantages:

- Reduction of slab thickness (material savings).
- Thinner slabs mean savings in time and labor.
- Reduction of deflections.
- Reduction of building height (cladding, mechanical, etc.)
- Reduction of building weight (foundation savings).
- Because prestressing steel has higher strength than mild steel, PT slabs are built with approximately 60 to 70% less steel (more space for MEP installations).
- Reduction of cracking due to permanent pre-compression, better corrosion resistant
- Architectural freedom of larger spans and irregular slab geometries
- Reduction in construction time since early stripping of formwork is permitted

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Pre-Stressing Construction Advantage

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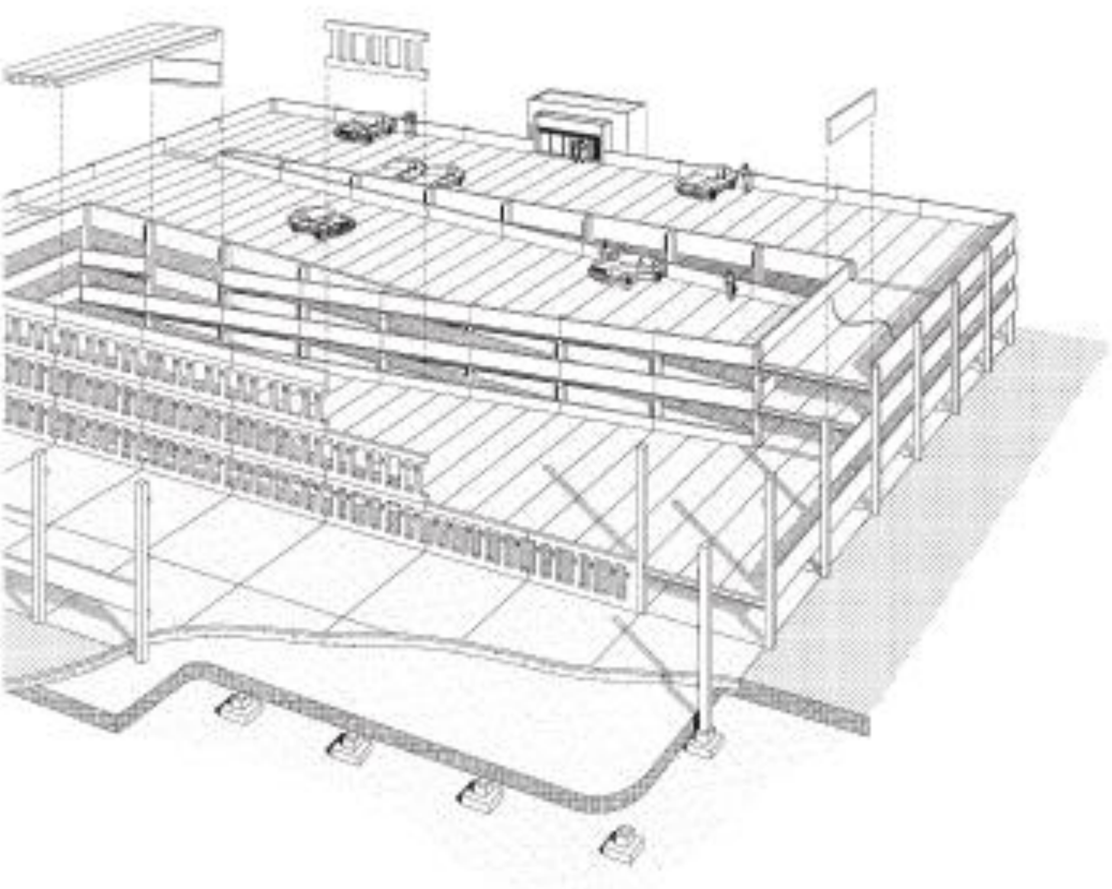
Pre-Stressing Construction Advantage



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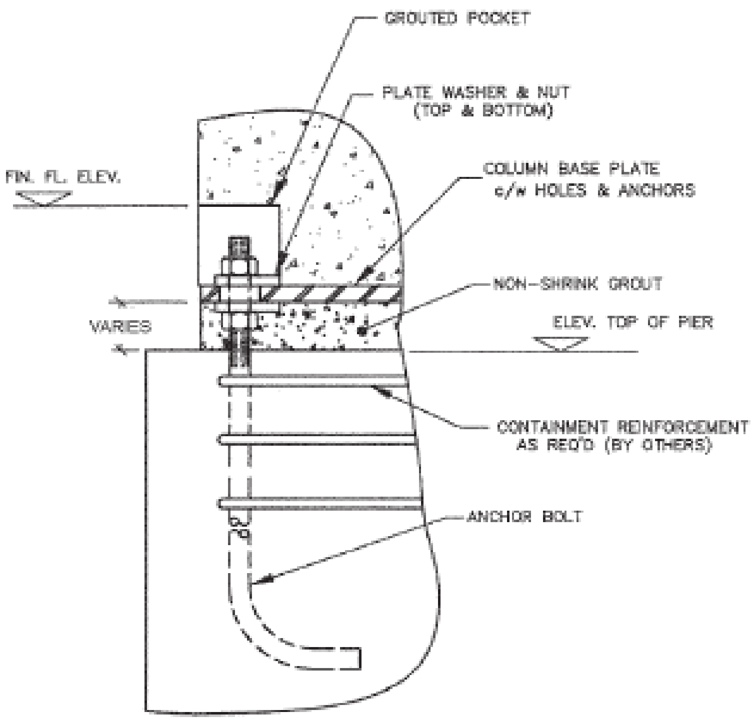
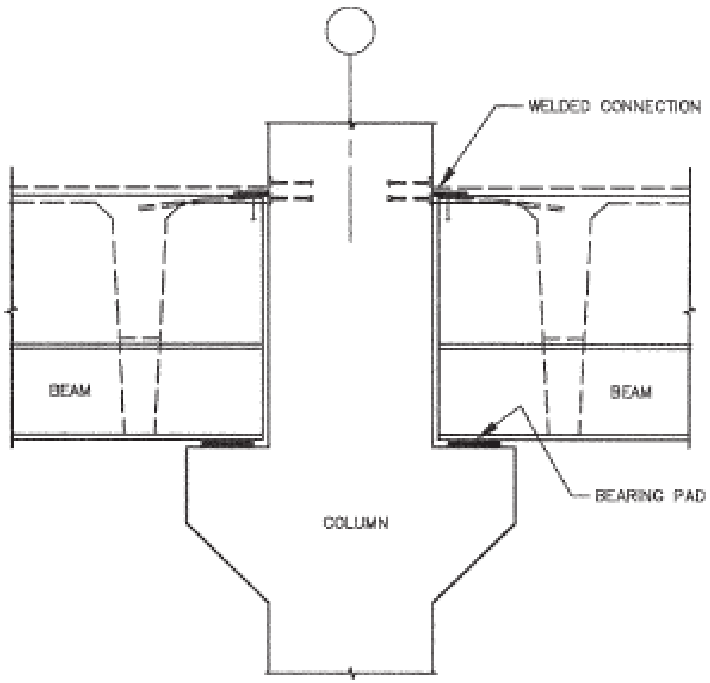
Pre-Stressing Element Connections





Inverted T/Beam to
Column Connection

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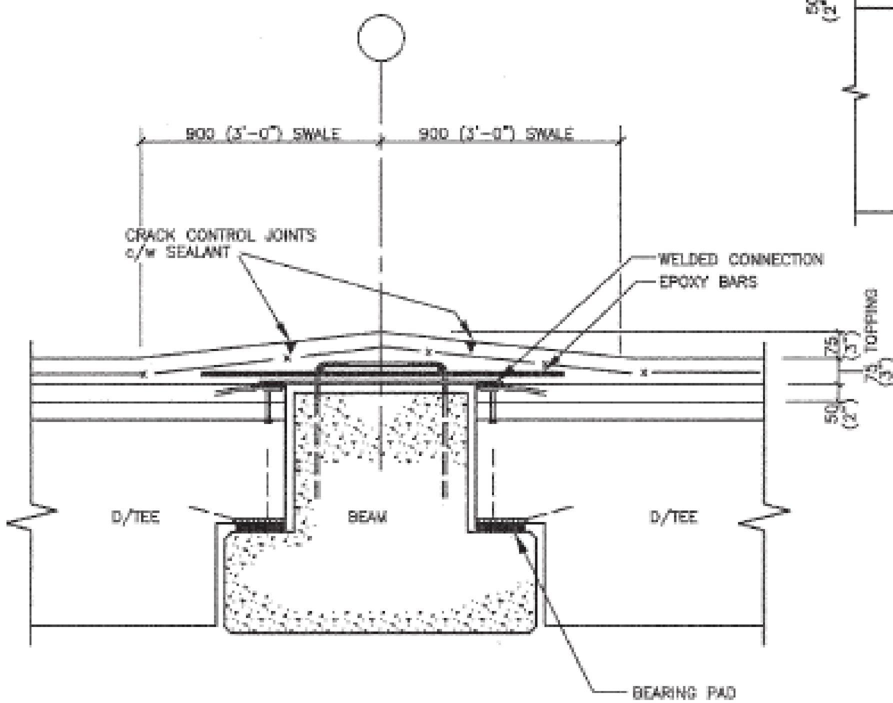
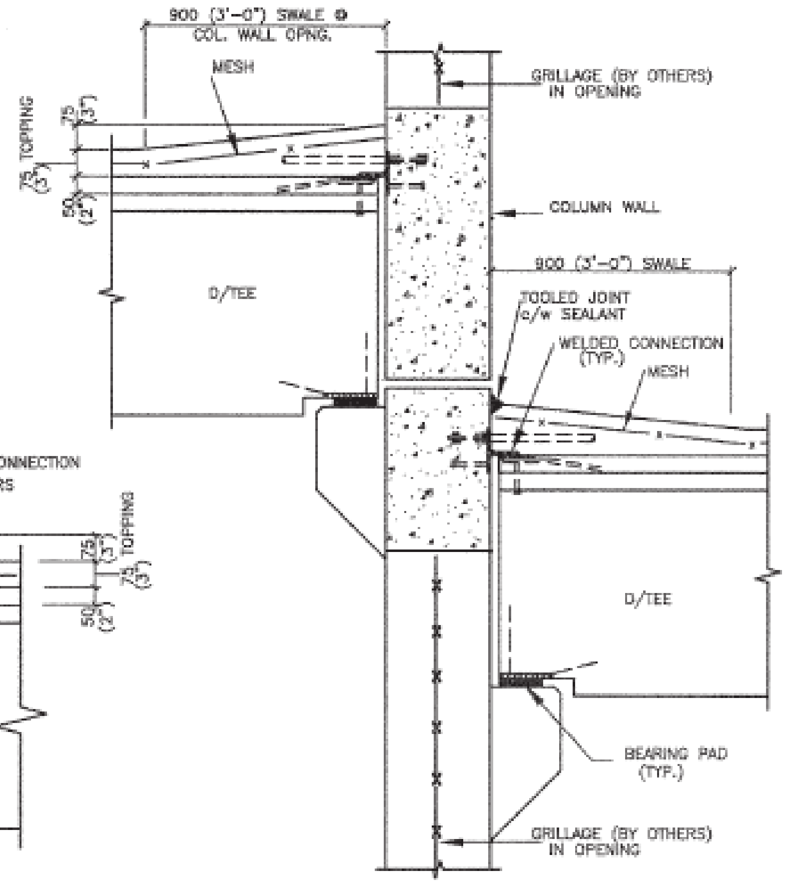
Pre-Stressing Element Connections



Inverted T/Beam to
Double Tee Connection



Column Wall to
Double Tee Connection

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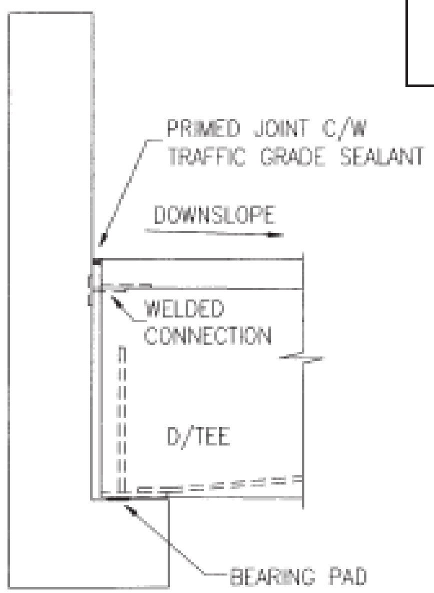
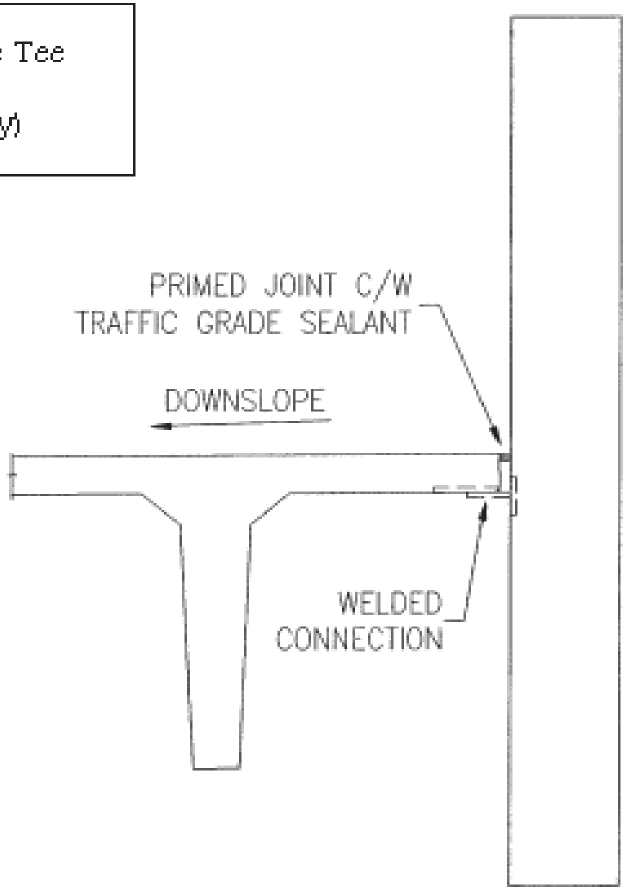
Pre-Stressing Element Connections

Pre-Topped Double Tee
Typical Details
(for concept only)



LB Spandrel to
Pre-Topped Double Tee
Connection



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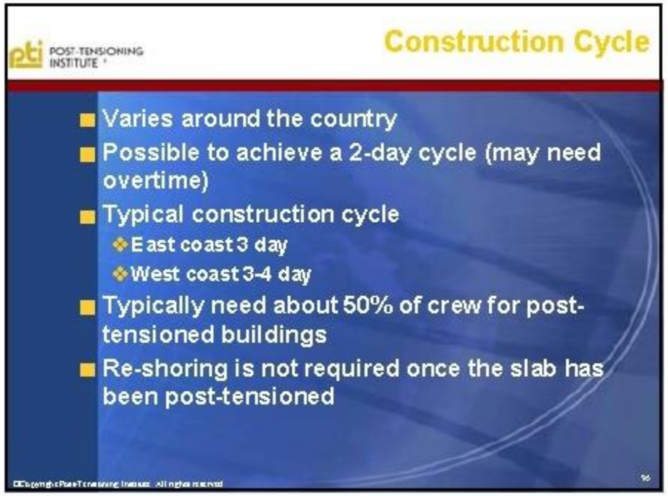
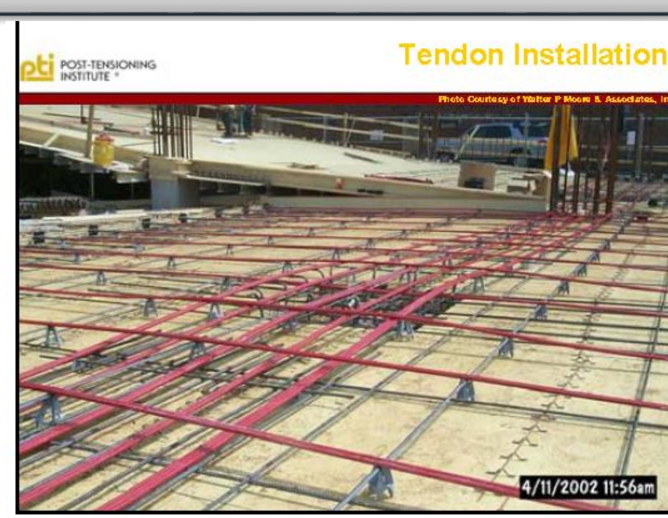
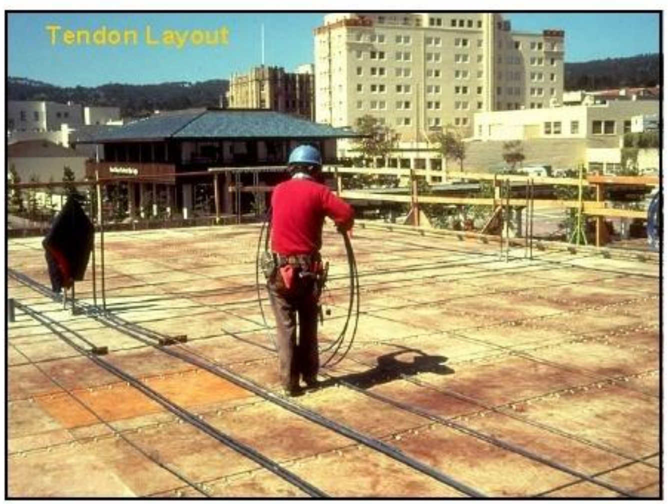
Pre-Stressed Slabs







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Pre-Stressed Slabs

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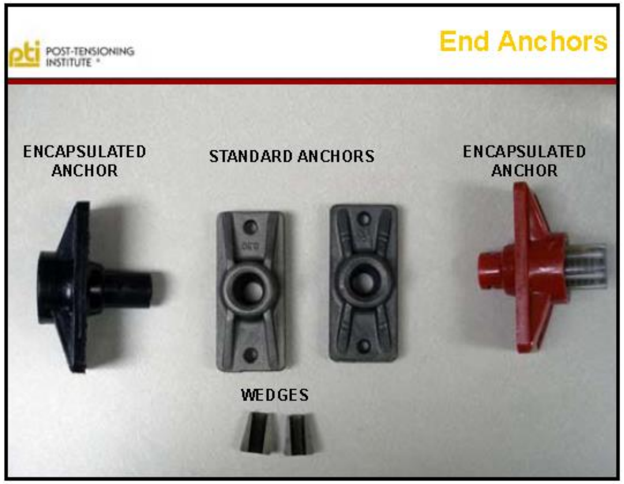

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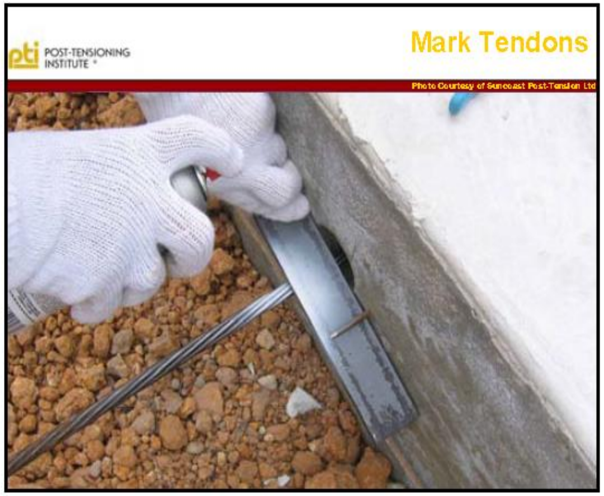
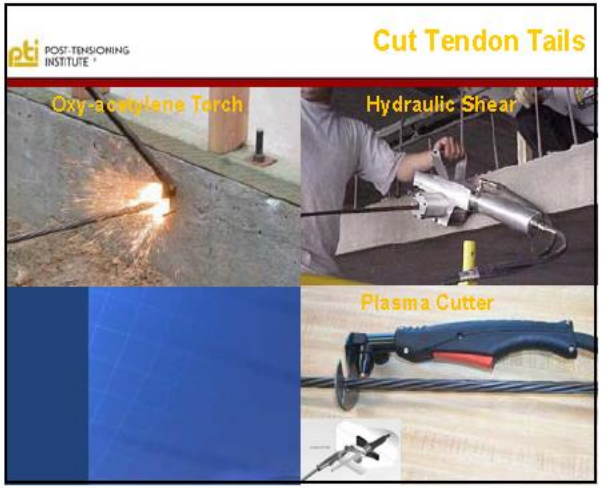
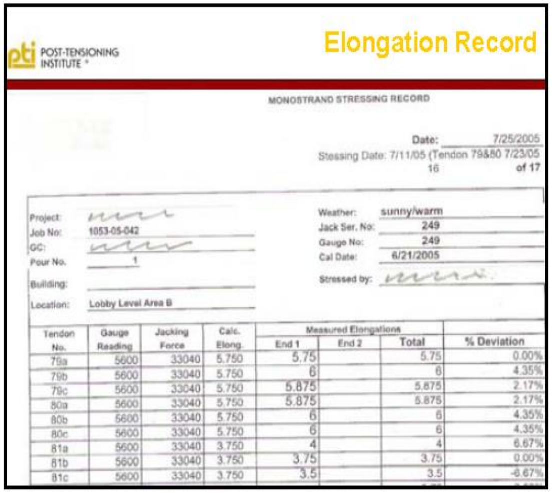





POST-TENSIONING INSTITUTE

Elongation Record

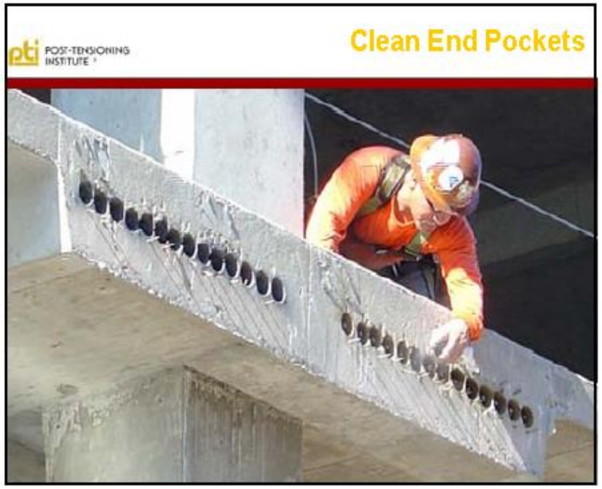
MONOSTRAND STRESSING RECORD

Date: 7/25/2005
Stessing Date: 7/11/05 (Tendon 79&80 7/23/05
16 of 17

Project:
Job No: 1053-05-042
GC:
Pour No. 1
Building:
Location: Lobby Level Area B

Weather: sunny/warm
Jack Ser. No: 249
Gauge No: 249
Cal Date: 6/21/2005
Stressed by:

Tendon No.	Gauge Reading	Jacking Force	Calc. Elong.	Measured Elongations End 1	End 2	Total	% Deviation
79a	5600	33040	5.750	5.75		5.75	0.00%
79b	5600	33040	5.750	6		6	4.35%
79c	5600	33040	5.750	5.875		5.875	2.17%
80a	5600	33040	5.750	5.875		5.875	2.17%
80b	5600	33040	5.750	6		6	4.35%
80c	5600	33040	5.750	6		6	4.35%
81a	5600	33040	3.750	4		4	6.67%
81b	5600	33040	3.750	3.75		3.75	0.00%
81c	5600	33040	3.750	3.5		3.5	-6.67%



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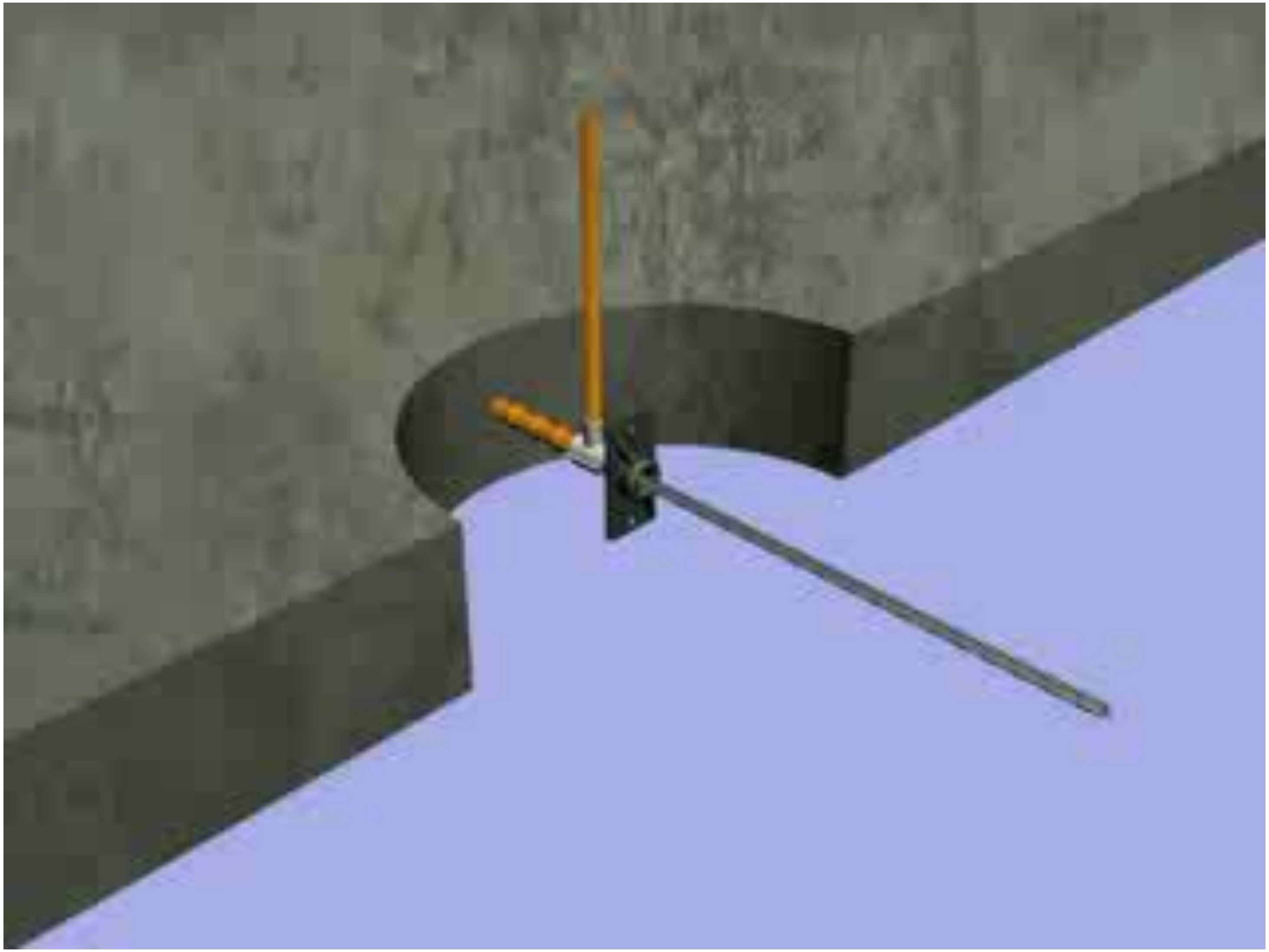
Pre-Stressed Slabs



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Pre-Stressed Slabs

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Pre-Stressed Beams

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Pre-Stressed Beams

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Pre-Stressed Columns

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Post-Tensioned Beams

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Post-Tensioned Beams

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Post-Tensioned Beams

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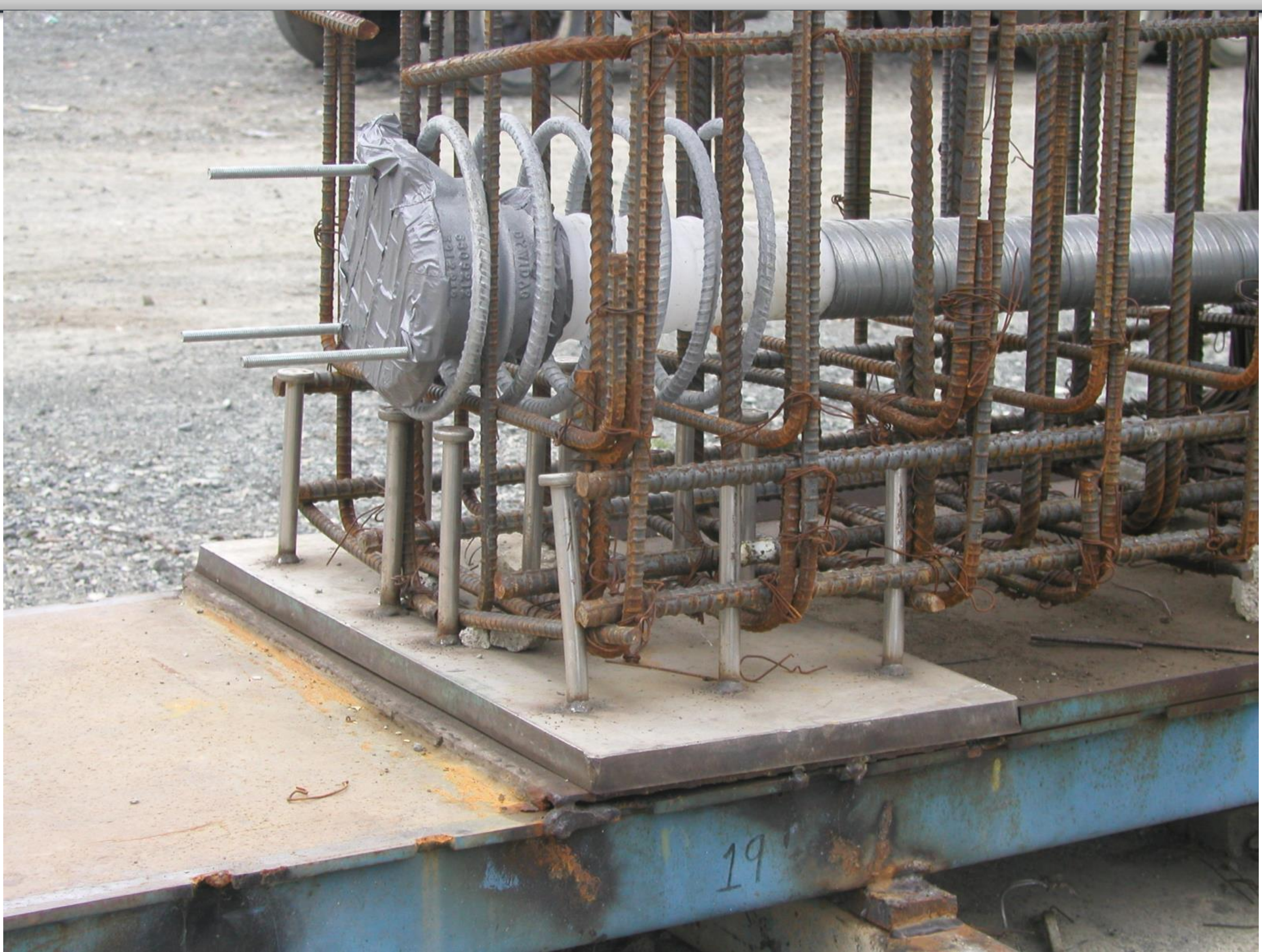
Post-Tensioned Beams



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Post-Tensioned Beams

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Post-Tensioned Beams

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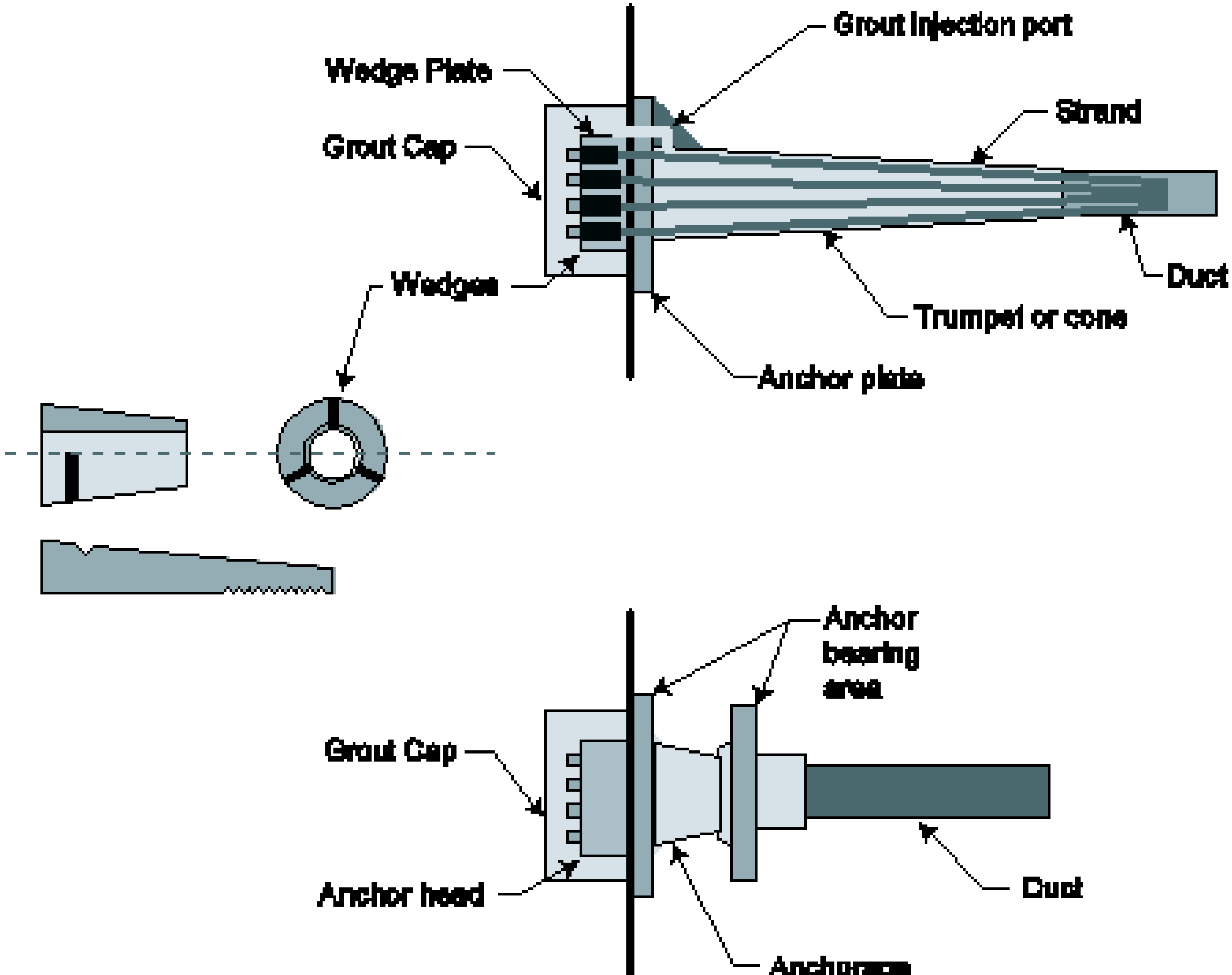
Post-Tensioned Anchorages



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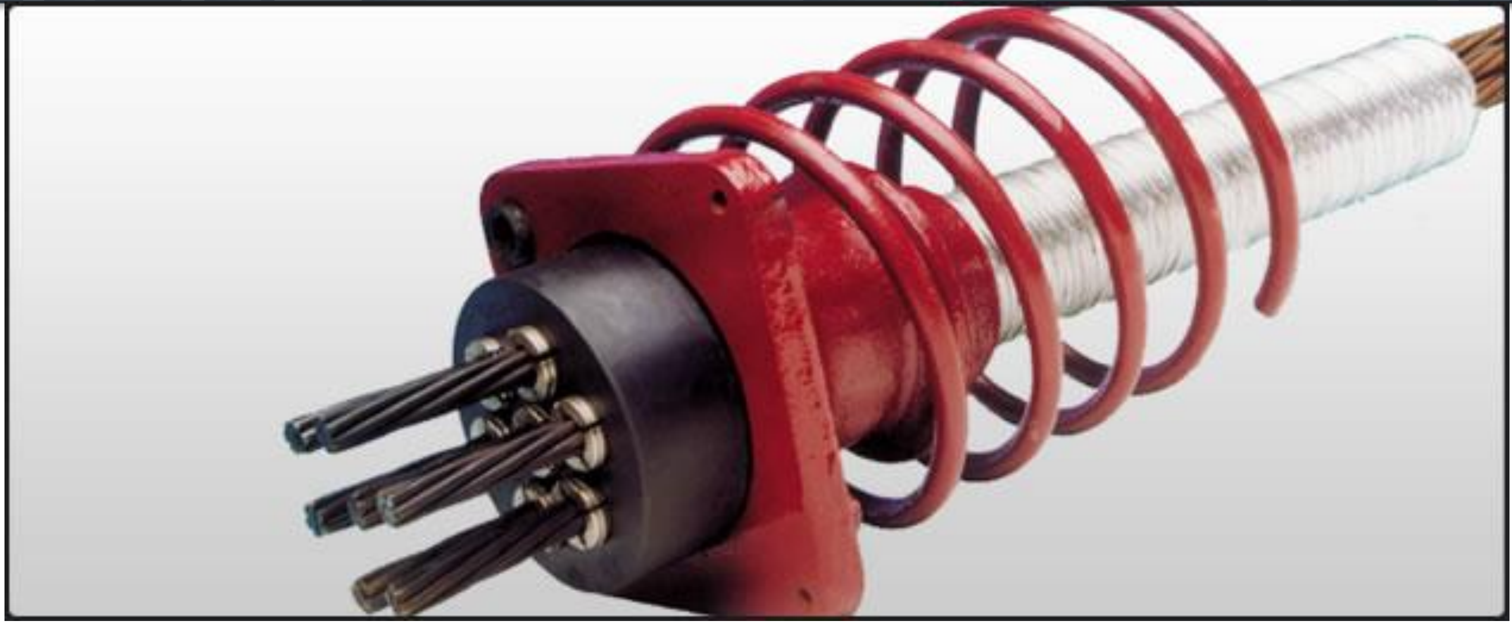
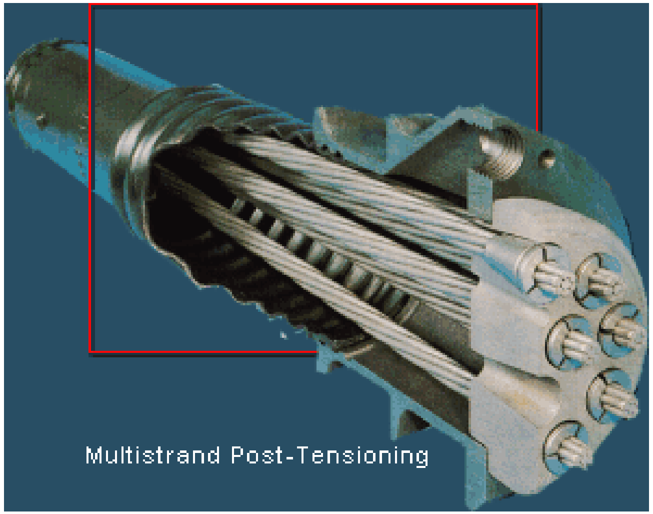
Post-Tensioned Anchorages





PRE-STRESSED CONCRETE SEMINAR

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Post-Tensioned Anchorages

PRE-STRESSED CONCRETE SEMINAR

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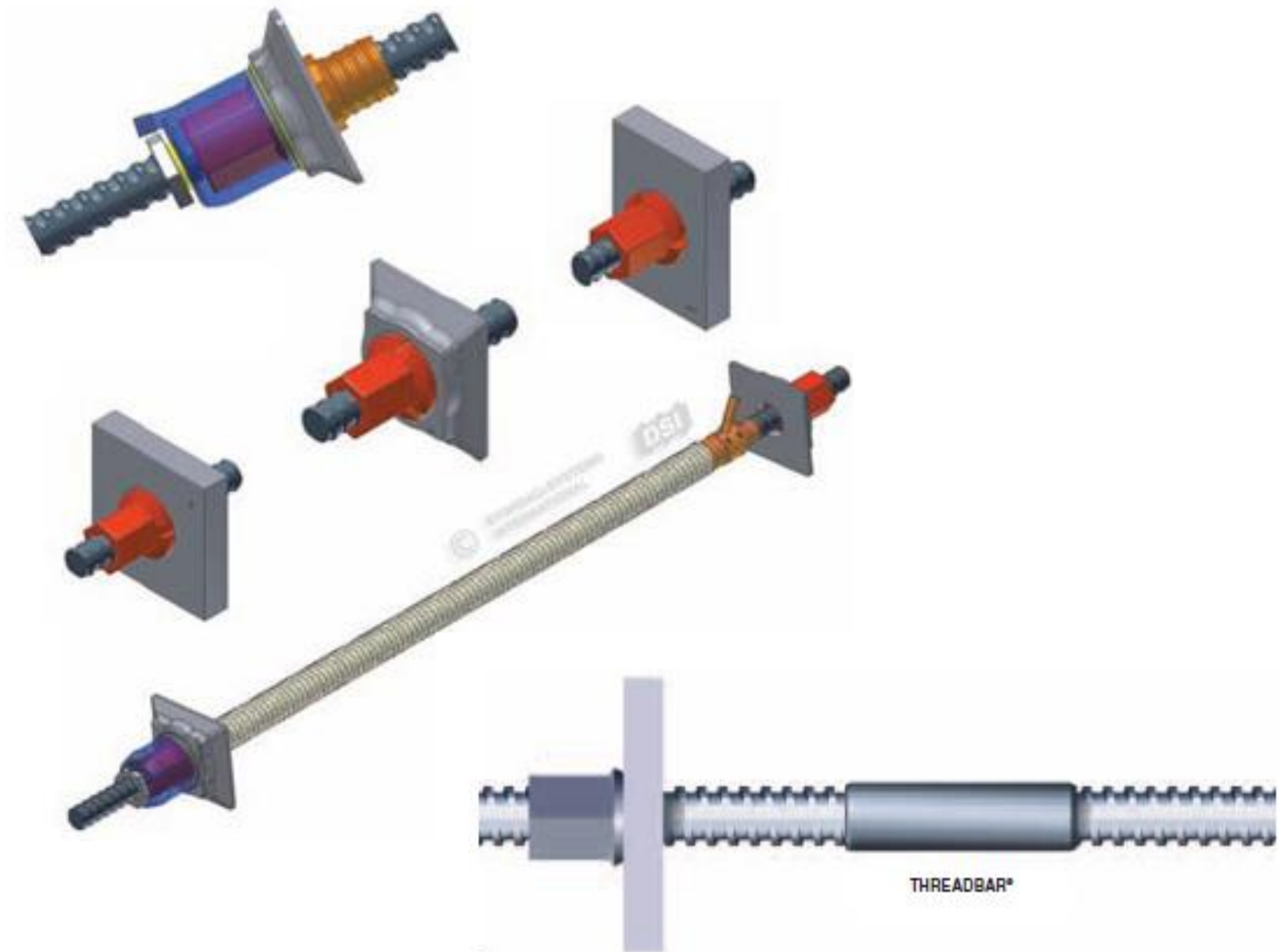
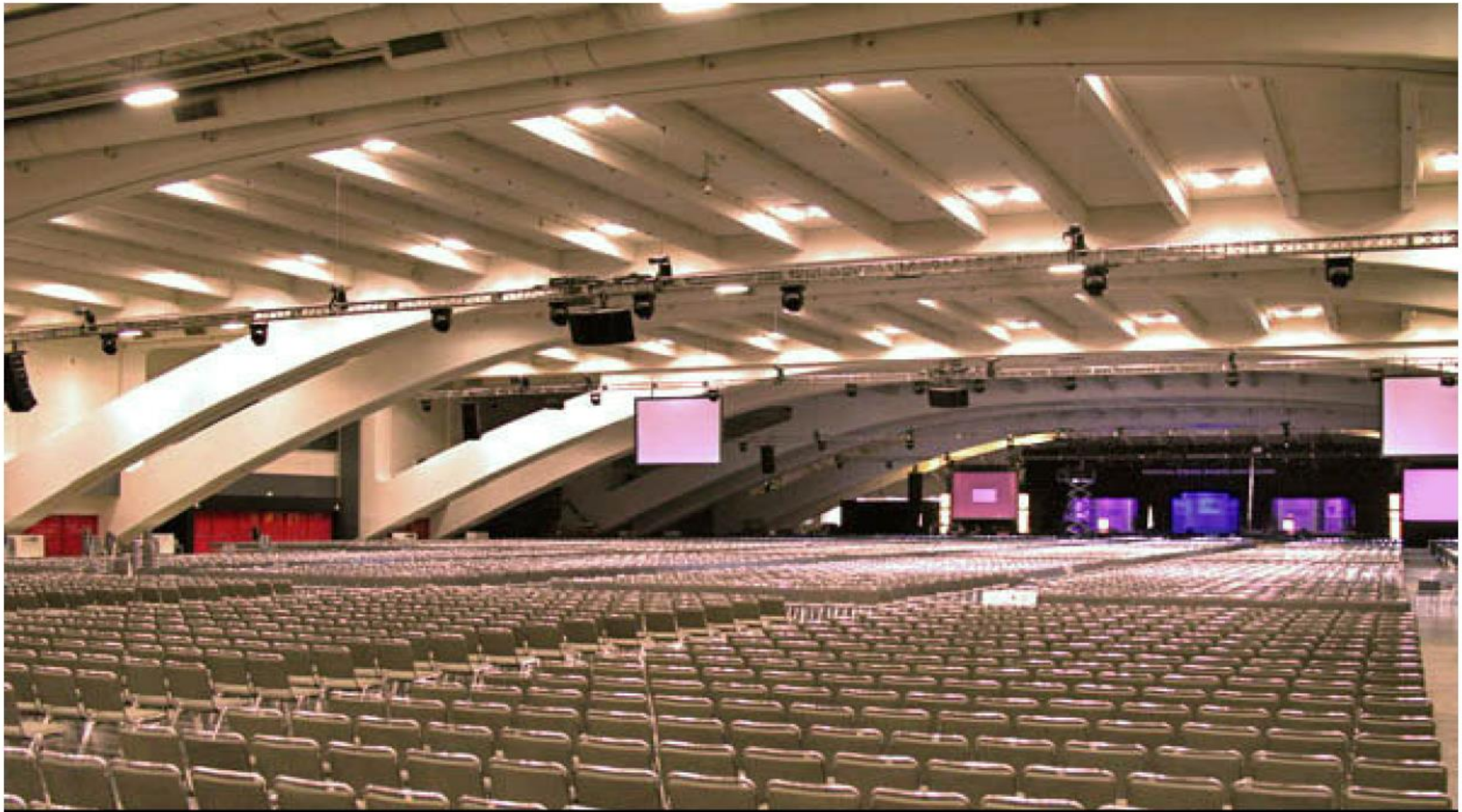
Post-Tensioned Bars



Pre-stressed Building Examples



Pre-stressed arches at the Moscone Center - San Francisco – 1981. Demonstrated effective use of prestressing to meet strict architectural guidelines (spans of 275 ft)

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Pre-stressed Building Examples

Juan Pachín Vicens Auditorium - sports venue in the city of Ponce, Puerto Rico – Designed by T.Y.Lin and built on 1972. 138 ft cantilevered.

ABC: Vigas Pretensadas para Puentes Principios Básicos de Diseño

Materials for Reinforced and Pre-stressed Concrete

PRE-STRESSED CONCRETE SEMINAR

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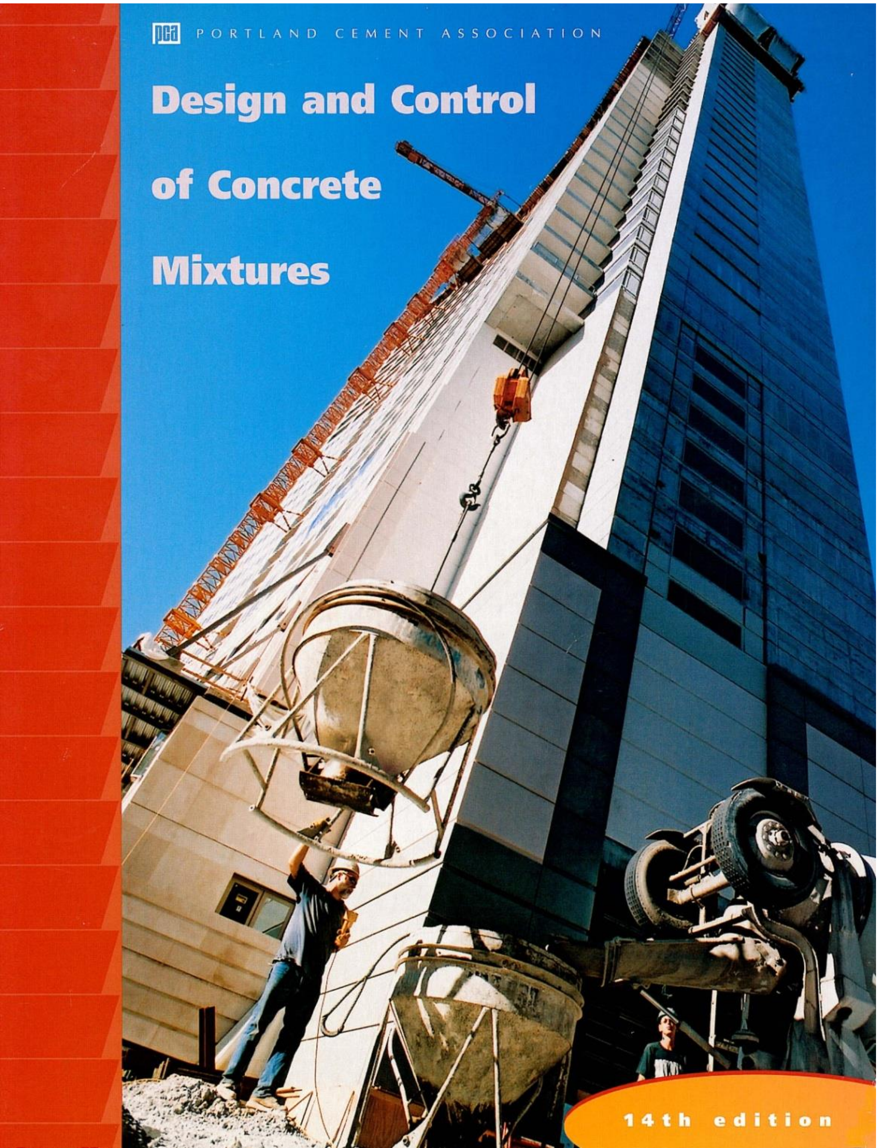
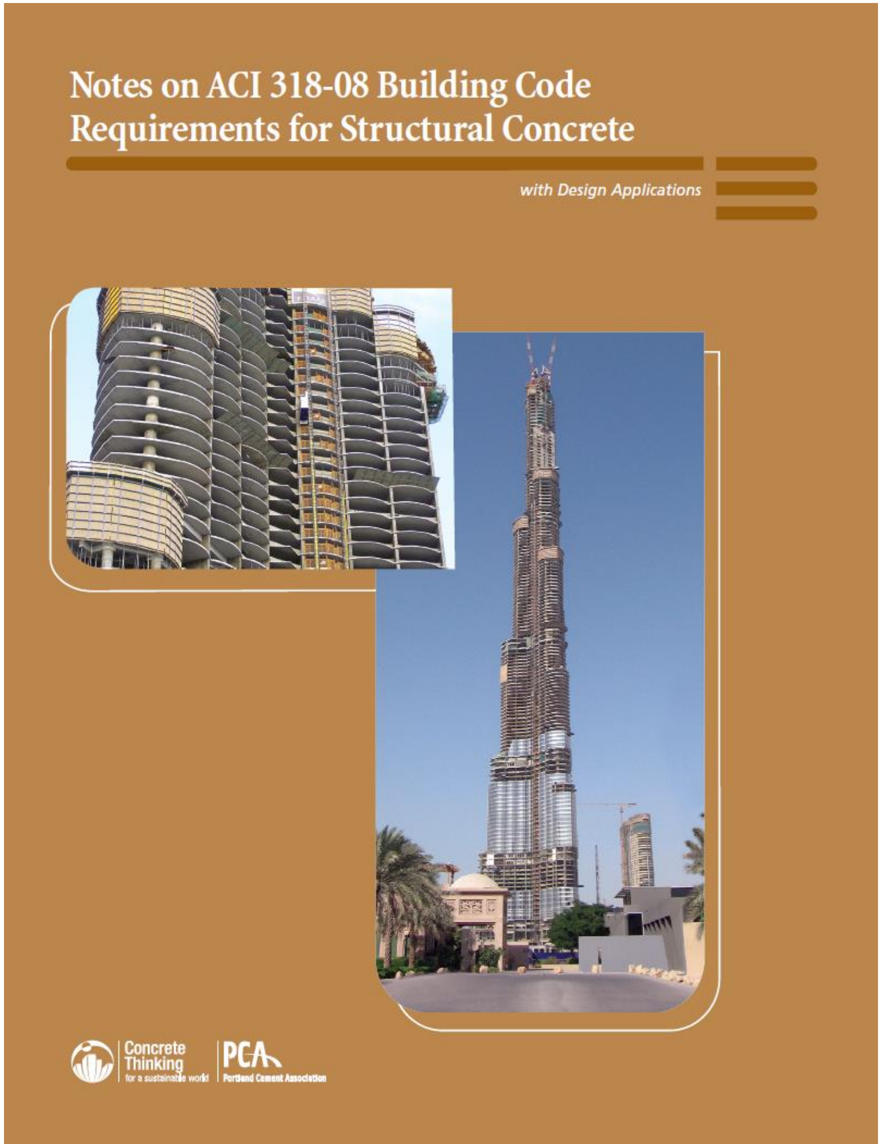
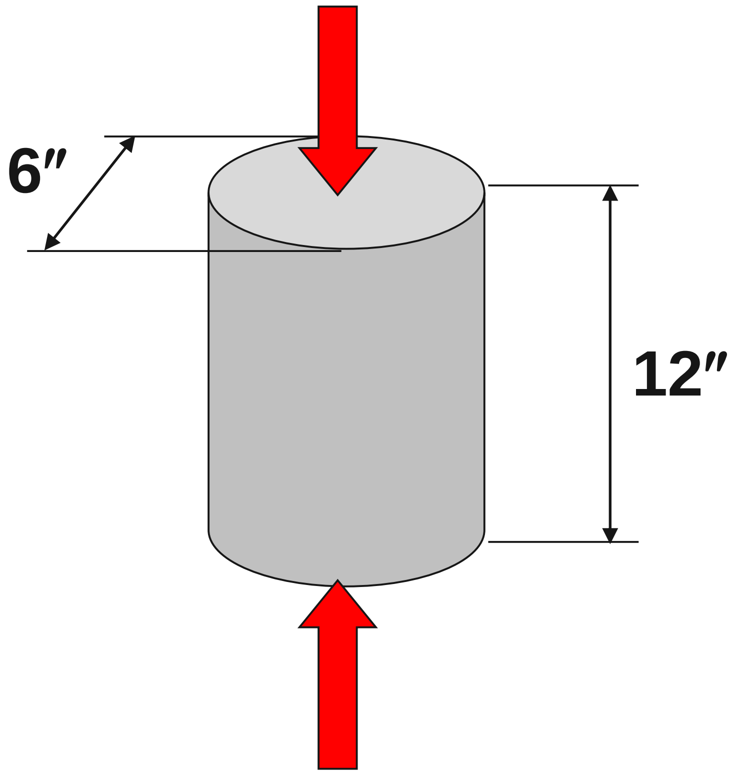
Presentation- References





Compressive Strength of Concrete

- Compressive strength



ASTM C 31

ASTM C 39

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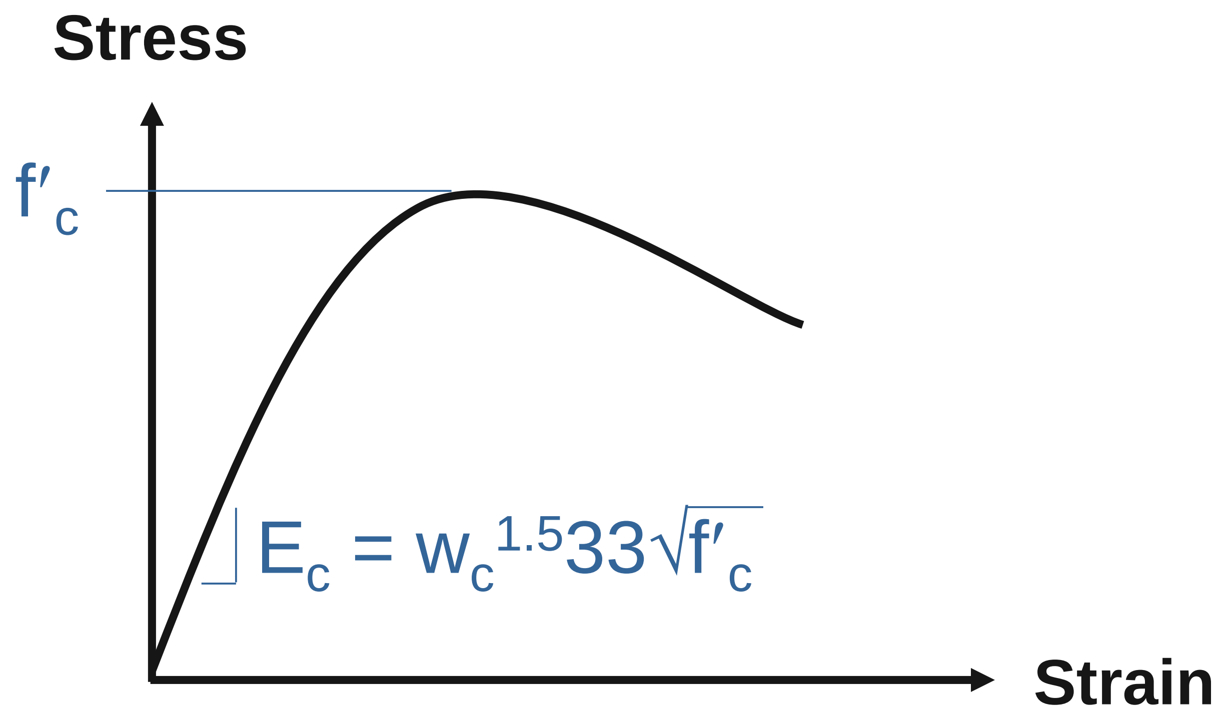
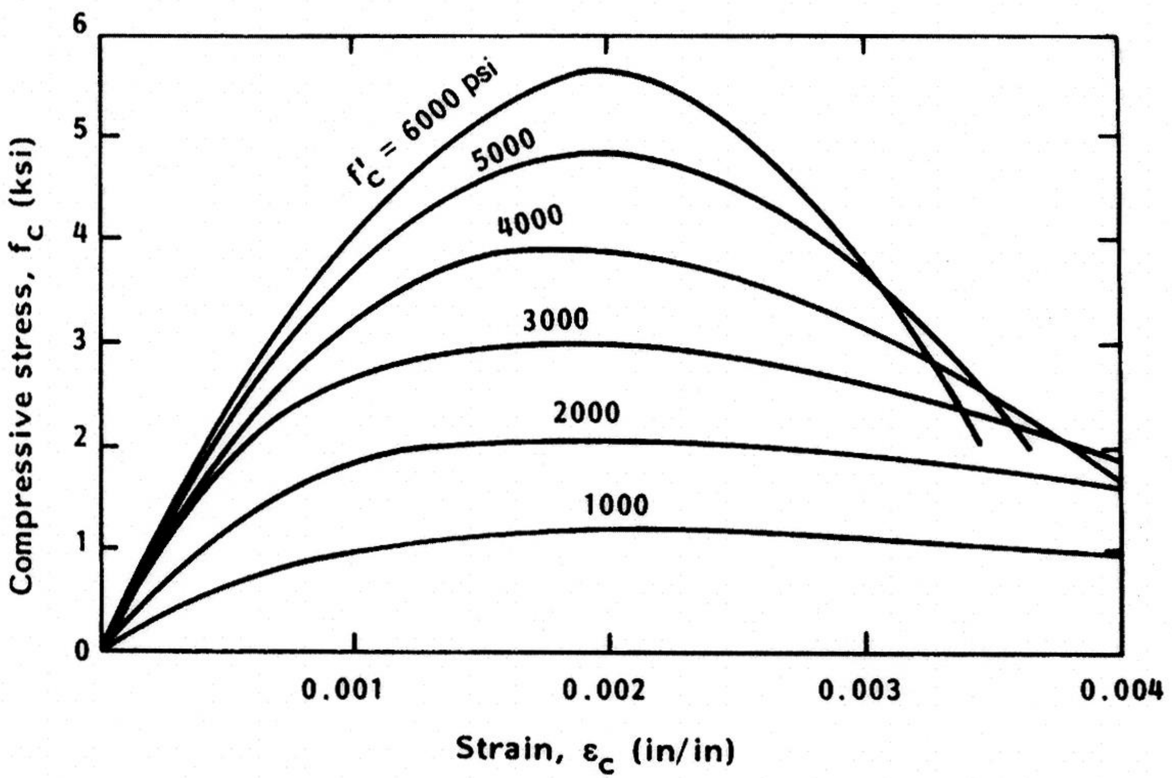
Compressive Strength of Concrete

- Compressive strength Curve



Compressive Strength of Concrete

- Compressive strength Curves



Tension Strength of Concrete

- Tensile strength
  - Varies between 8% and 15% of the compressive strength

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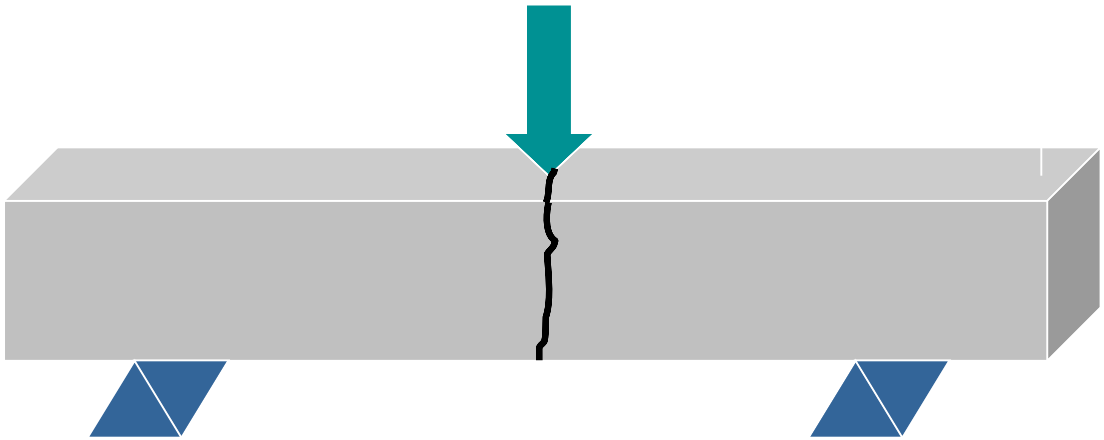
Tension Strength of Concrete

- Modulus of rupture (flexural test)

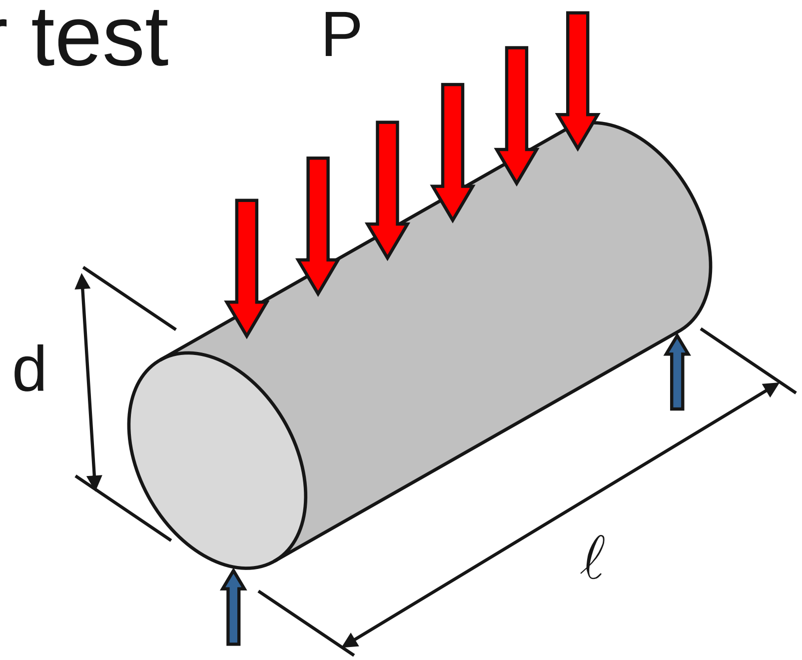


Tension Strength of Concrete

- Modulus of rupture (flexural test)
  - ASTM C 78 – Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)
  - ASTM C 293 – Standard Test Method for Flexural Strength of Concrete (Using Simple Beam With Center-Point Loading)

Tension Strength of Concrete

- Split cylinder test



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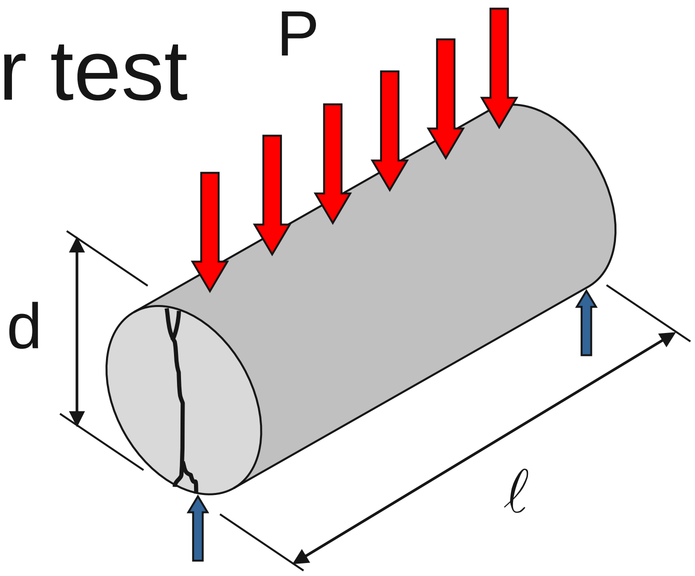
Tension Strength of Concrete

- Split cylinder test



$$f_{ct} = \frac{2P}{\pi \ell d}$$

Tension Strength of Concrete

- Split cylinder test
  - ASTM C 496 – Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens

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Tension and Compression Strength Relationship

- Relationship between compressive and tensile strengths
  - Tensile strength increases with an increase in compressive strength
  - Ratio of tensile strength to compressive strength decreases as the compression strength increases
  - Tensile strength $\propto \sqrt{f'_c}$

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Tension and Compression Strength Relationship

- Relationship between compressive and tensile strengths
  - Mean $f_{ct} = 6.4\sqrt{f'_c}$
  - For deflections (Eq. 9-10):
    - $f_r = 7.5\sqrt{f'_c}$
  - For strength (ACI 11.4.3.1):
    - $f_r = 6\sqrt{f'_c}$

Shrinkage, Creep and Thermal Expansion

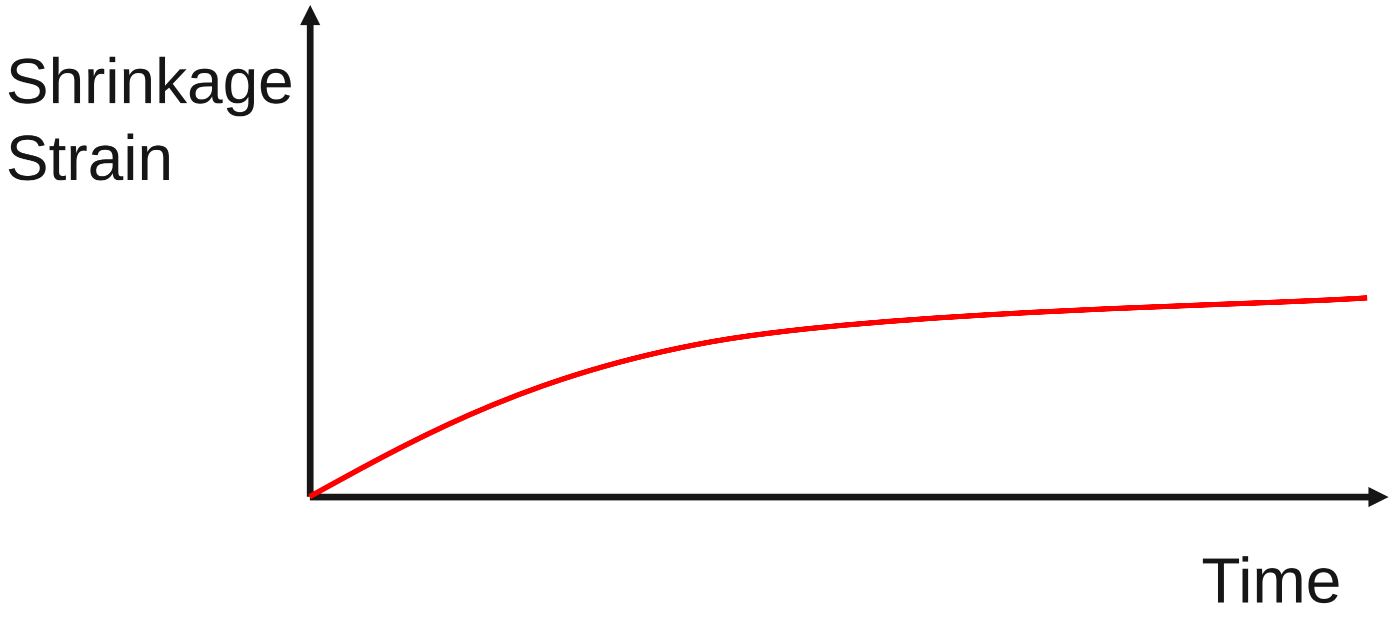
- Shrinkage
- Creep
- Thermal expansion

Shrinkage

- Shrinkage
  - Shortening of concrete during hardening and drying under constant temperature
    - Moisture diffuses out of the concrete
    - Exterior shrinks more than the interior
    - Tensile stresses in the outer layer and compressive stresses in the interior

Shrinkage

- Shrinkage

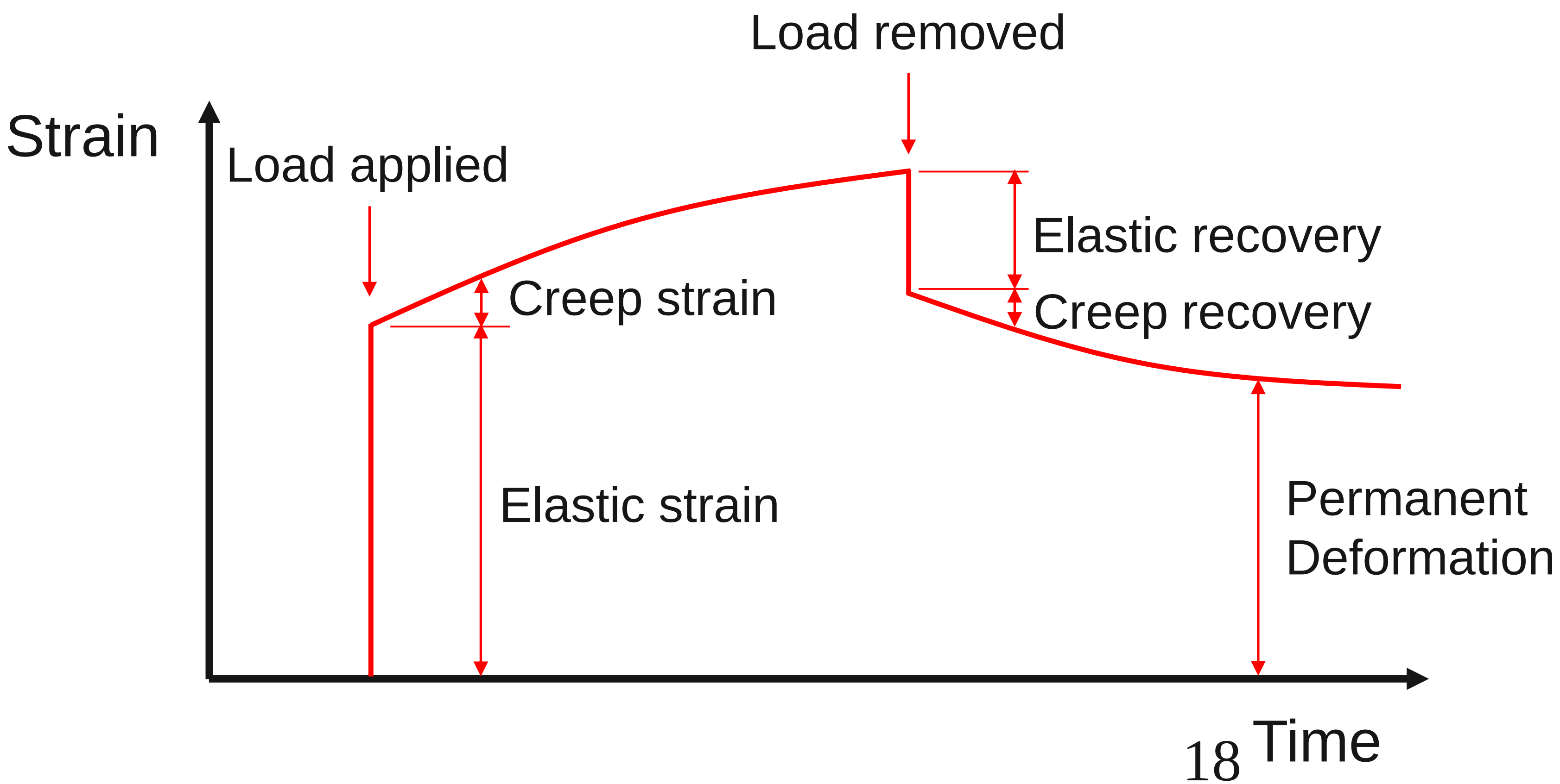


Shrinkage

- Shrinkage
  - When not adequately controlled, can cause:
    - Unsightly or harmful cracks
    - Large and harmful stresses
    - Partial loss of initial prestress
  - Reinforcement restrains the development of shrinkage

Creep

- Creep


Load removed
Strain
Load applied
Elastic recovery
Creep strain
Creep recovery
Elastic strain
Permanent Deformation
Time

Creep

- Creep strains can lead to
  - Increase in deflections with time
  - Redistribution of stresses
  - Decrease in pre-stressing forces

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Thermal Expansion

- Thermal expansion
  - Coefficient of thermal expansion or contraction
    - Affected by:
      - Composition of the concrete
      - Moisture content of the concrete
      - Age of the concrete

Thermal Expansion

- Thermal expansion
  - Coefficient of thermal expansion or contraction
    - Normal weight concrete
      - Siliceous aggregate: 5 to 7 x 10^{-6} strain/°F
      - Limestone/calcareous aggregate: 3.5 to 7 x 10^{-6} strain/°F
    - Lightweight concrete
      - 3.6 to 6.2 x 10^{-6} strain/°F
    - A value of 5.5 x 10^{-6} strain/°F is satisfactory

Reinforcing Steels

- Deformed Bar Reinforcement
- Welded Wire Reinforcement
- Pre-stressing Steel

Deformed Bar Reinforcement

- ASTM A 615, Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement
- ASTM A 706, Specification for Low-Alloy Steel Deformed and Plain Bars for Concrete Reinforcement
- ASTM A 996, Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement

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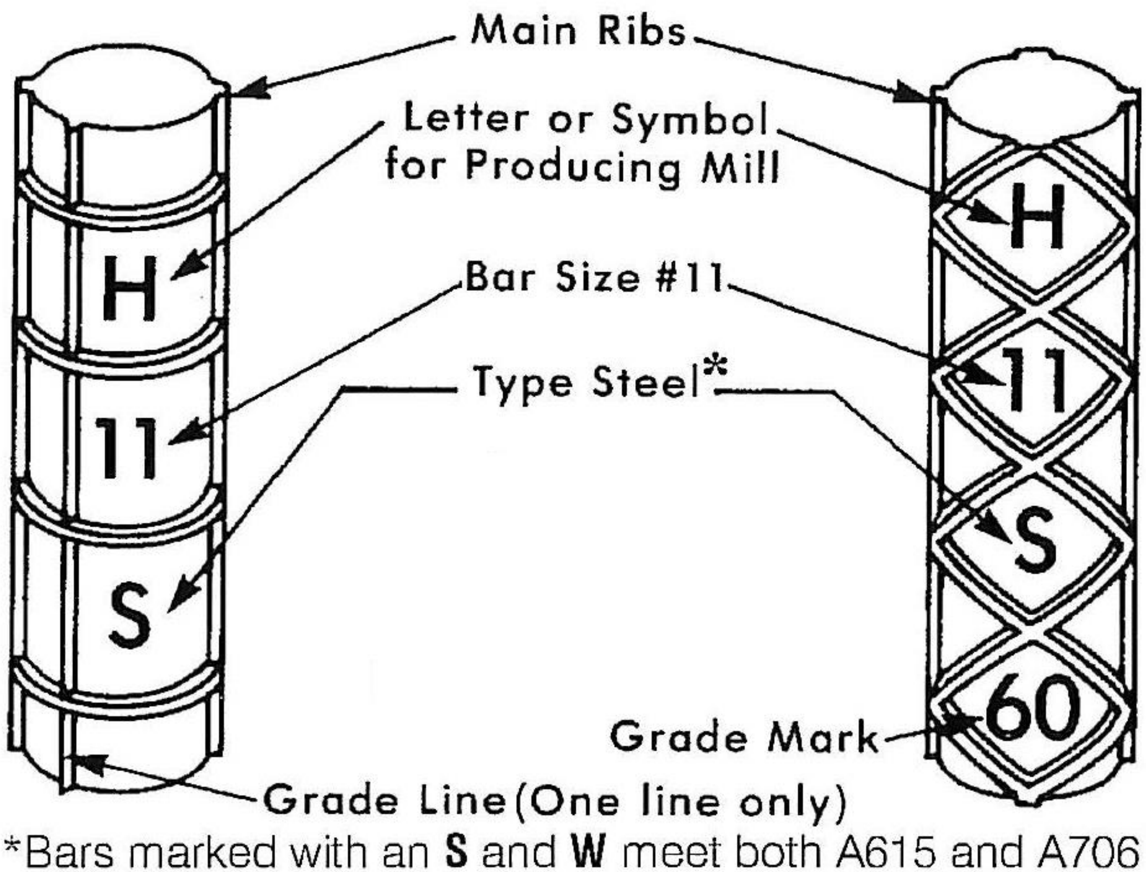
Deformed Bar Reinforcement



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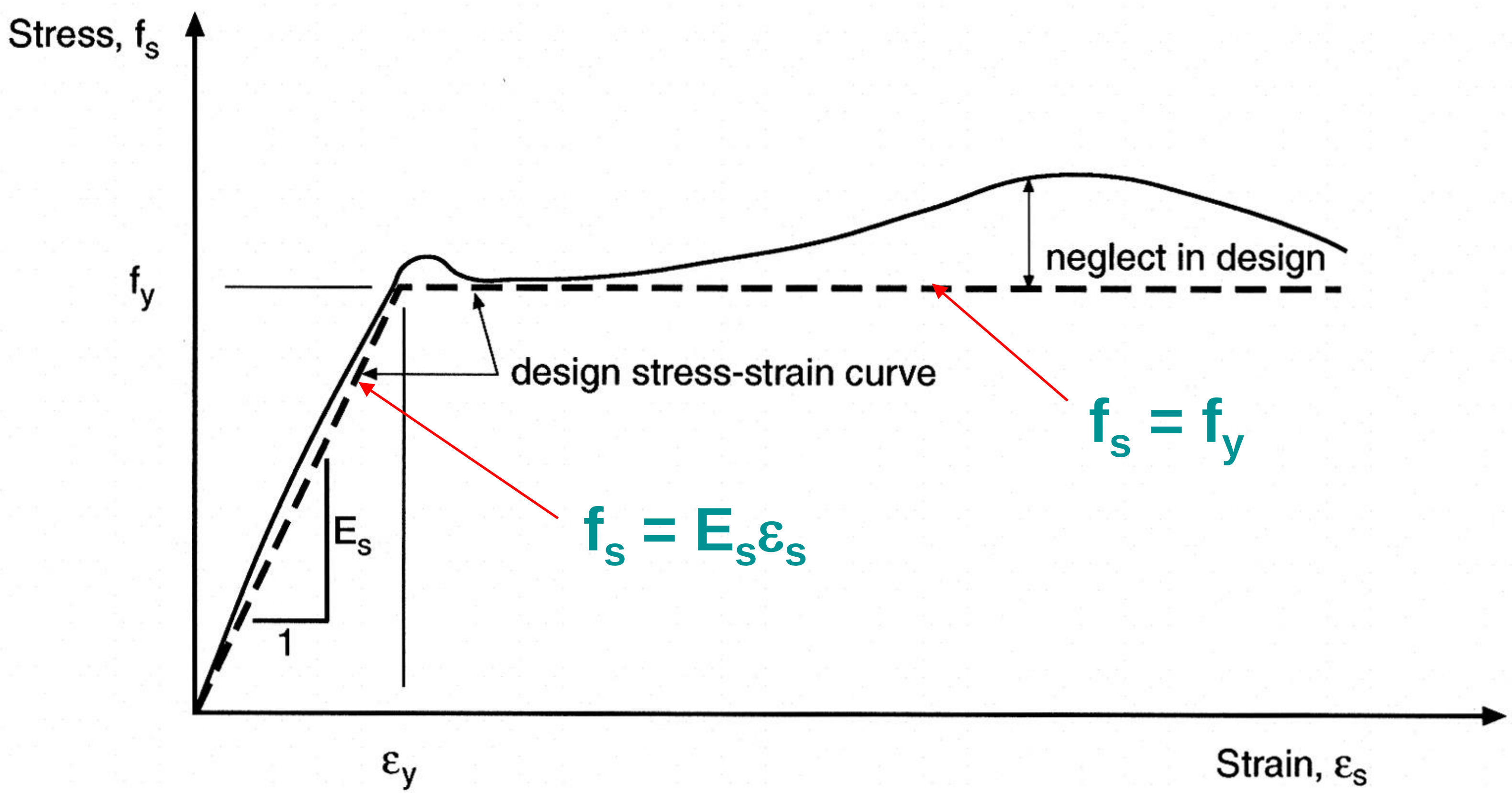
Deformed Bar Reinforcement



Deformed Bar Reinforcement

Bar Designation	Area (in.^{2})	Weight (plf)	Diameter (in.)
No. 3	0.11	0.376	0.375
No. 4	0.20	0.668	0.500
No. 5	0.31	1.043	0.625
No. 6	0.44	1.502	0.750
No. 7	0.60	2.044	0.875
No. 8	0.79	2.670	1.000
No. 9	1.00	3.400	1.128
No. 10	1.27	4.303	1.270
No. 11	1.56	5.313	1.410
No. 14	2.25	7.650	1.693
No. 18	4.00	13.600	2.257

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Pre-stressing Steel

- Strands
  - Seven-wire
  - Three- and Four-wire
- Wire
- Bars
  - Plain
  - Deformed

Pre-stressing Steel

- ASTM A 416, Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete
- ASTM A 421, Standard Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete
- ASTM A 722, Standard Specification for Uncoated High-Strength Steel Bars for Prestressing Concrete

Pre-stressed 7-wire strand

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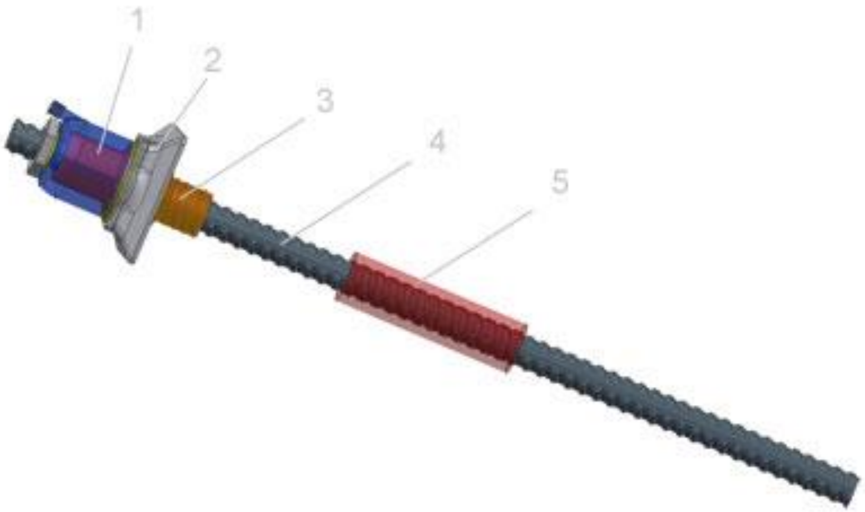

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Prestressing Steel

- 7-Wire Strands, $f_{pu} = 270$ ksi

Nominal Diameter (in.)	Area (in.^{2})	Weight (plf)
3/8	0.085	0.29
7/16	0.115	0.40
1/2	0.153	0.52
9/16	0.192	0.65
3/5	0.217	0.74

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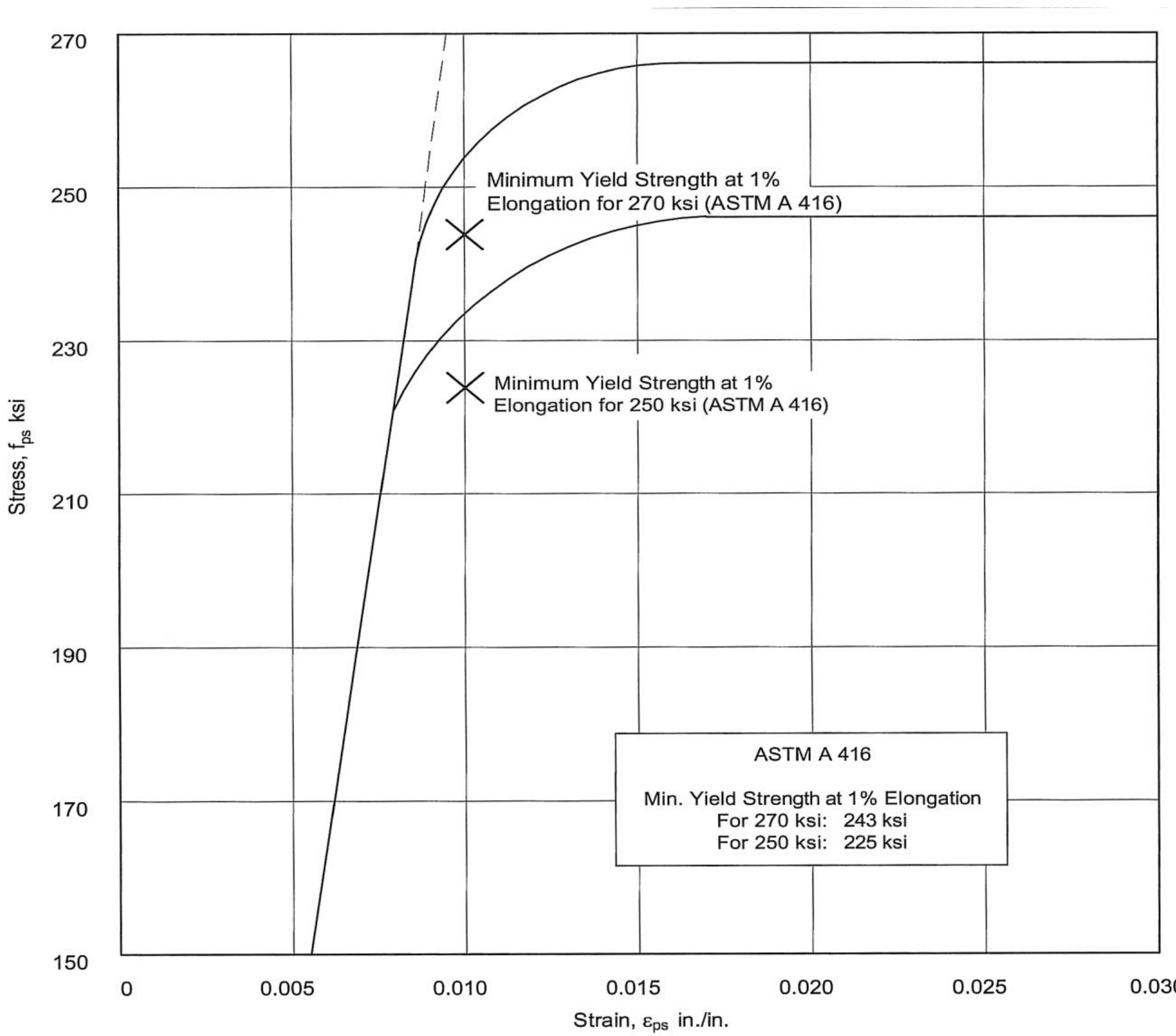
Prestressing Steel



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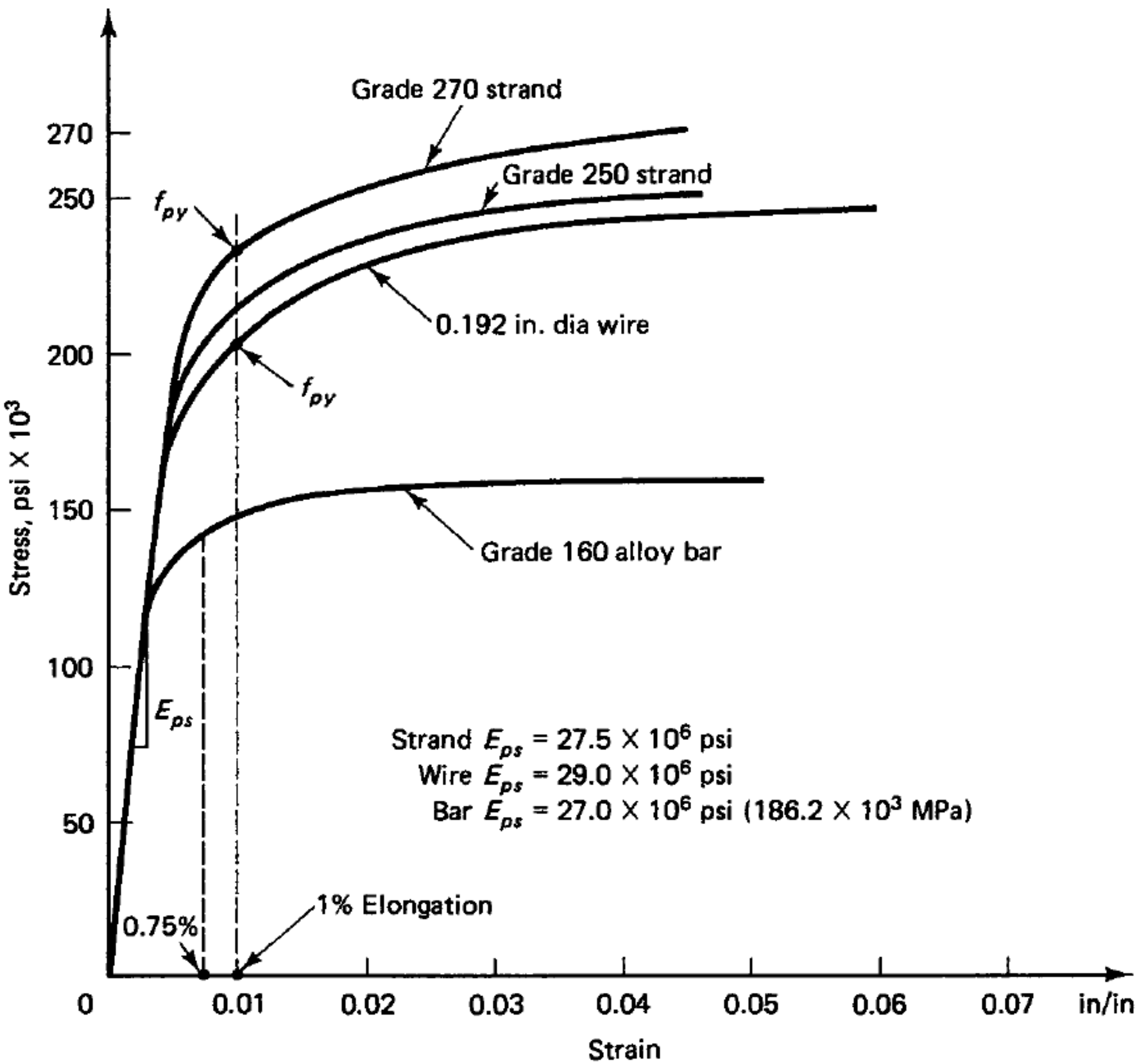
Prestressing Steel



Prestressing Steel

SEVEN-WIRE GRADE 270 LOW-RELAXATION STRANDS					
Nominal Diameter of Strand (in)	Nominal Area of Strand A_s^* (in^2)	Ultimate Strength of Strand $f_s' \times A_s^*$ (lbs)	Yield Strength of Strand $f_y^* \times A_s^*$ (lbs)	$0.75 \times f_s' \times A_s^*$ (lbs)	$0.90 \times f_y^* \times A_s^*$ (lbs)
3/8 (0.375)	0.085	22950	20660	17210	18590
7/16 (0.438)	0.115	31050	27950	23290	25160
1/2 (0.500)	0.153	41310	37180	30980	33460
1/2 (special)	0.167	45000	40500	33750	36450
0.60	0.217	58590	52730	43940	47460

The $^1/_2$" (special) strand shown above is not listed in ASTM A416. It is a $^1/_2$" diameter strand designed to have a minimum breaking strength of 45,000 lbs. and a minimum yield strength of 40,500 lbs.

$f_y^* = 0.90 \times f_s'$ = yield stress of prestressing steel (AASHTO 9.1.2).

$0.75 \times f_s' \times A_s^*$ = required tensioning force per strand immediately prior to release (after losses due to anchorage set and other factors). This force shall be entered in the table on the standard beam detail sheet as the "Prestress Force Per Strand" (AASHTO 9.15.1).

$0.90 \times f_y^* \times A_s^*$ = maximum tensioning force per strand for short periods of time prior to seating to offset seating and friction losses (AASHTO 9.15.1).

Losses in Pre-stressed Concrete

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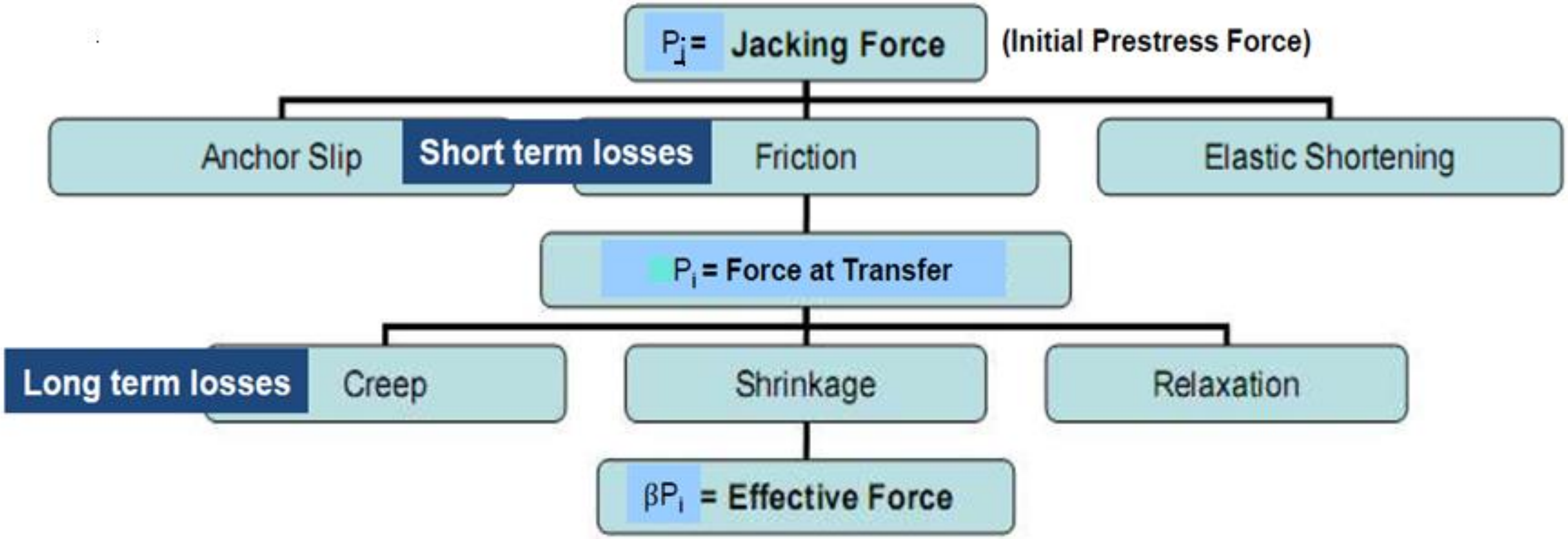
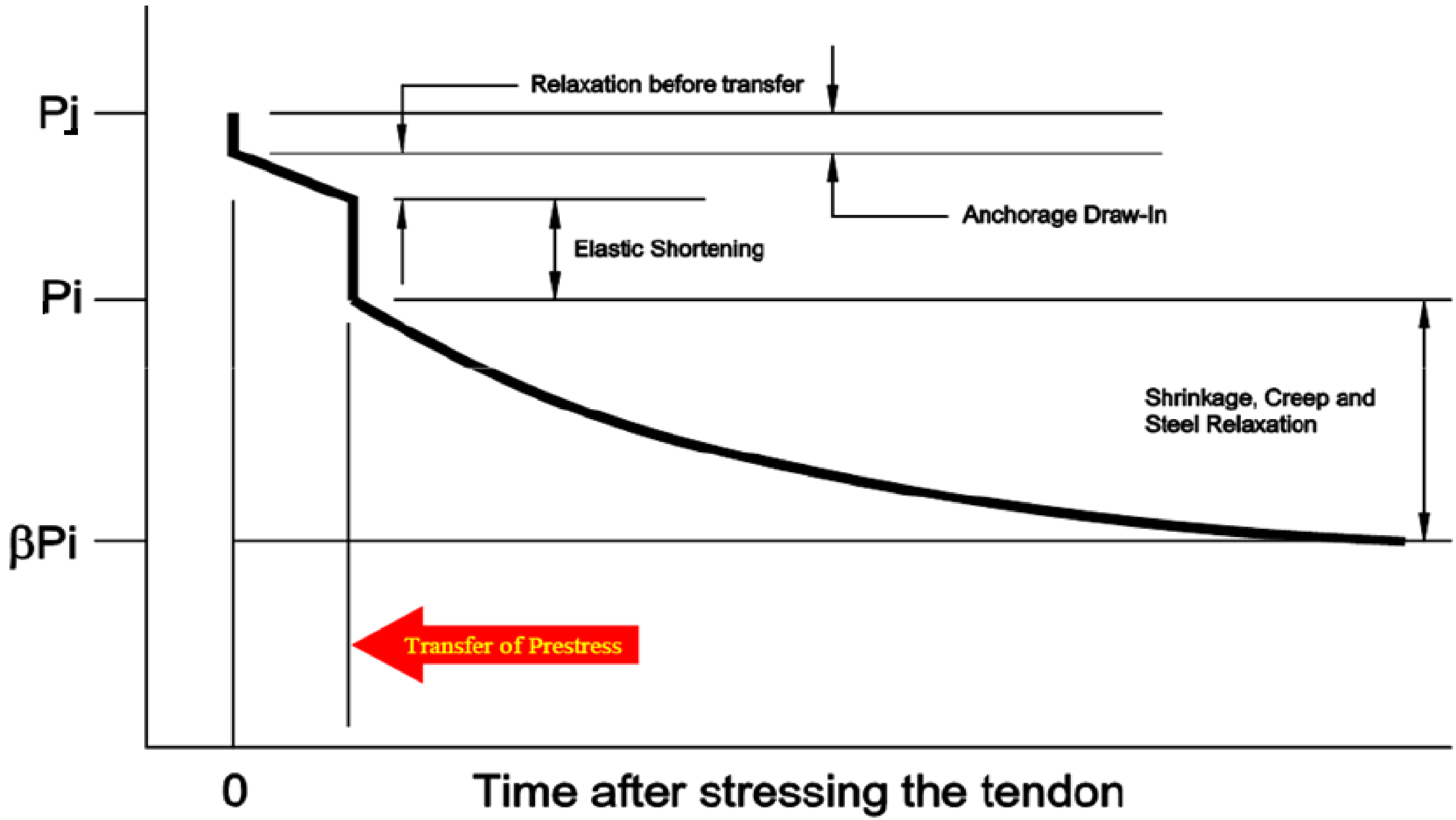
Pre-stressed Losses

The fraction of the jacking force (Pj) that is eventually transferred to the concrete after releasing the temporary anchor or withdrawal of the hydraulic jack is the Pi.

This force also keeps on decreasing with time due to time-dependent response of constituent materials; steel and concrete and reduced to a final value known as effective pre-stressing force, βPi.



Pre-stressed Losses



Topics Addressed

- Pre-stressed Losses
  - Elastic Shortening
  - Creep of Concrete
  - Shrinkage of Concrete
  - Steel Relaxation

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Pre-stressed Losses

Essentially, the reduction in the pre-stressing force can be grouped into two categories:
Immediate elastic loss during the fabrication or construction process, including elastic shortening of the concrete, anchorage losses, and frictional losses.

Time-dependent losses such as creep and shrinkage and those due to temperature effects and steel relaxation, all of which are determinable at the service-load limit state of stress in the pre-stressed concrete element.

An exact determination of the magnitude of these losses, particularly the time-dependent ones, is not feasible, since they depend on a multiplicity of interrelated factors. Empirical methods of estimating

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Pre-stressed Losses

losses differ with the different codes of practice or recommendations, such as those of the Pre-stressed Concrete Institute, the ACI–ASCE joint committee approach, the AASHTO lump-sum approach, the Comite´ Eurointernationale du Be´ton (CEB), and the FIP (Federation Internationale de la Pre´contrainte). The degree of rigor of these methods depends on the approach chosen and the accepted practice of record

NOTATION

A_{ps} = *total area of pre-stressing steel*

e = *eccentricity of center of gravity of pre-stressing steel with respect to center of gravity of concrete at the cross-section considered*

E_c = *modulus of elasticity of concrete at 28 days*
E_{ci} = *modulus of elasticity of concrete at time prestress is applied*
E_s = *modulus of elasticity of prestressing steel. Usually 28,500,000 psi*

f_{cds} = *stress in concrete at center of gravity of prestressing steel due to all superimposed permanent dead loads that are applied to the member after it has been pre-stressed*
f_{cir} = *net compressive stress in concrete at center of gravity of prestressing steel immediately after the pre-stress has been applied to the concrete.*
f_{cpa} = *average compressive stress in the concrete along the member length at the center of gravity of the pre-stressing steel immediately after the prestress has been applied to the concrete*

NOTATION (Cont.)

f_g = *stress in concrete at center of gravity of prestressing steel due to weight of structure at time prestress is applied*

f_{pi} = *stress in prestressing steel due to Ppi = Ppi/Aps*

f_{pu} = *specifi ed tensile strength of prestressing steel, psi*

I_c = *moment of inertia of gross concrete section at the cross-section considered*

M_d = *bending moment due to dead weight of member being prestressed and to any other permanent loads in place at time of prestressing*

M_{ds} = *bending moment due to all superimposed permanent dead loads that are applied to the member after it has been prestressed*

P_{pi} = *prestressing force in tendons at critical location on span after reduction for losses due to friction and seating loss at anchorages but before reduction for ES, CR, SH, and RE*

Elastic Shortening of Concrete (ES)

For members with bonded tendons:

$ES = K_{es}\ E_s\ f_{cir}/E_{ci}$ (1)

where K_{es} = 1.0 for pre-tensioned members

K_{es} = 0.5 for post-tensioned members where tendons are tensioned in sequential order to the same tension.

With other post-tensioning procedures, the value for Kes may vary from 0 to 0.5.

$f_{cir} = K_{cir}\ f_{cpi} - fg$ (2)

Where: K_{cir} = 1.0 for post-tensioned members

K_{cir} = 0.9 for pre-tensioned members

For members with bonded tendons:

$ES = K_{es}\ E_s\ f_{cpa}/E_{ci}$ (1a)

Creep Loss (CR)

In general, this loss is a function of the stress in the concrete at the section being analyzed. In posttensioned, nonbonded members, the loss can be considered essentially uniform along the whole span. Hence, an average value of the concrete stress between the anchorage points can be used for calculating the creep in posttensioned members.

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Creep of Concrete (CR)

In general, this loss is a function of the stress in the concrete at the section being analyzed. In post-tensioned, non-bonded members, the loss can be considered essentially uniform along the whole span. Hence, an average value of the concrete stress between the anchorage points can be used for calculating the creep in post-tensioned members.

For members with bonded tendons:

$CR = K_{cr} E_s * (f_{cir} - f_{cds}) / E_c$ (3)

Where: K_{cr} = 2.0 for pre-tensioned members

K_{cr} = 1.6 for post-tensioned members

For members made of sand-lightweight concrete the foregoing values of Kcr should be reduced by 20 percent.

For members with unbonded tendons:

$CR = K_{cr} E_s / E_c * f_{cpa}$ (3a)

Shrinkage of Concrete (SH)

As with concrete creep, the magnitude of the shrinkage of concrete is affected by several factors. They include mixture proportions, type of aggregate, type of cement, curing time, time between the end of external curing and the application of pre-stressing, size of the member, and the environmental conditions. Size and shape of the member also affect shrinkage. Approximately 80% of shrinkage takes place in the first year of life of the structure. The average value of ultimate shrinkage strain in both moist-cured and steam-cured concrete is given in the ACI 209 R-92 Report.

For post-tensioned members, the loss in pre-stressing due to elastic shortening and shrinkage is somewhat less since most of the elastic shortening and some shrinkage has already taken place before post-tensioning.

Shrinkage of Concrete (SH)

$SH = 8.2\ 10\text{-}6\ K_{sh} E_s\ (1 - 0.06\ V/S)\ (100 - RH)$ (4)

where K_{sh} = 1.0 for pretensioned members

K_{sh} is taken from Table for post-tensioned members.

TABLE 1

Values of K_{sh} for Post-Tensioned Members

Time, days*	1	3	5	7	10	20	30	60
K_{sh}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

**Time after end of moist curing to application of prestress*

Relaxation of Tendons (RE)

$$RE = [Kre - J\ (SH + CR + ES)]\ C \qquad (5)$$

where the values of K_{re}, J, and C are taken from Tables 2 and 3.

TABLE 2

Values of K_{re} and J

Type of Tendon	K_{re} (psi)	J
270 Grade stress-relieved strand or wire	20,000	0.15
250 Grade stress-relieved strand or wire	18,500	0.14
240 or 235 Grade stress-relieved wire	17,600	0.13
270 Grade low-relaxation strand	5000	0.040
250 Grade low-relaxation wire	4630	0.037
240 or 235 Grade low-relaxation wire	4400	0.035
145 or 160 Grade stress-relieved bar	6000	0.05

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Relaxation of Tendons (RE)

TABLE 3

Values of C

f_{pi}/f_{pu}	Stress relieved strand or wire	Stress relieved low relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

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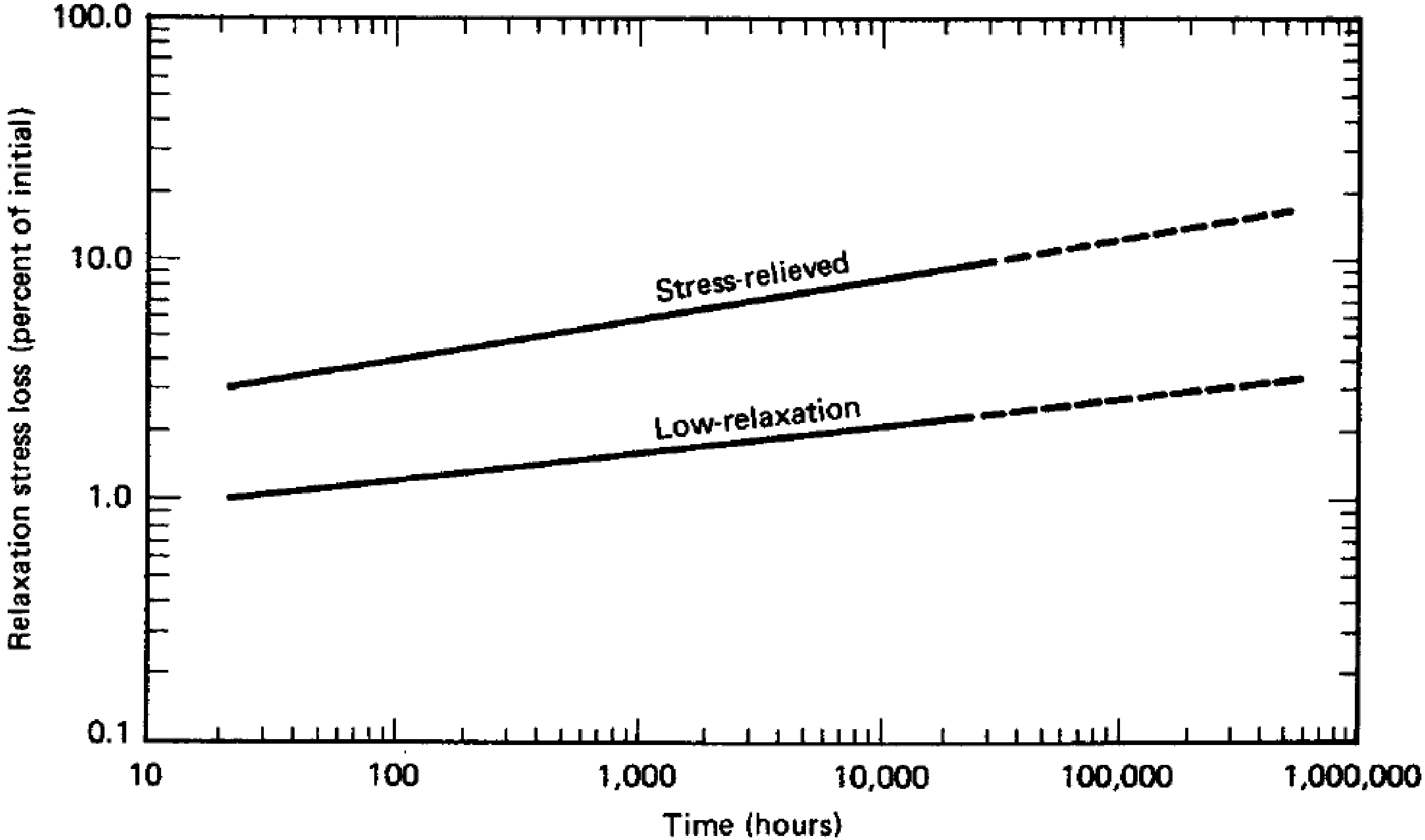
Relaxation of Tendons (RE)



Relaxation loss vs. time for stress-relieved low-relaxation prestressing steels at 70 percent of the ultimate. (*Courtesy*, Post-Tensioning Institute.)

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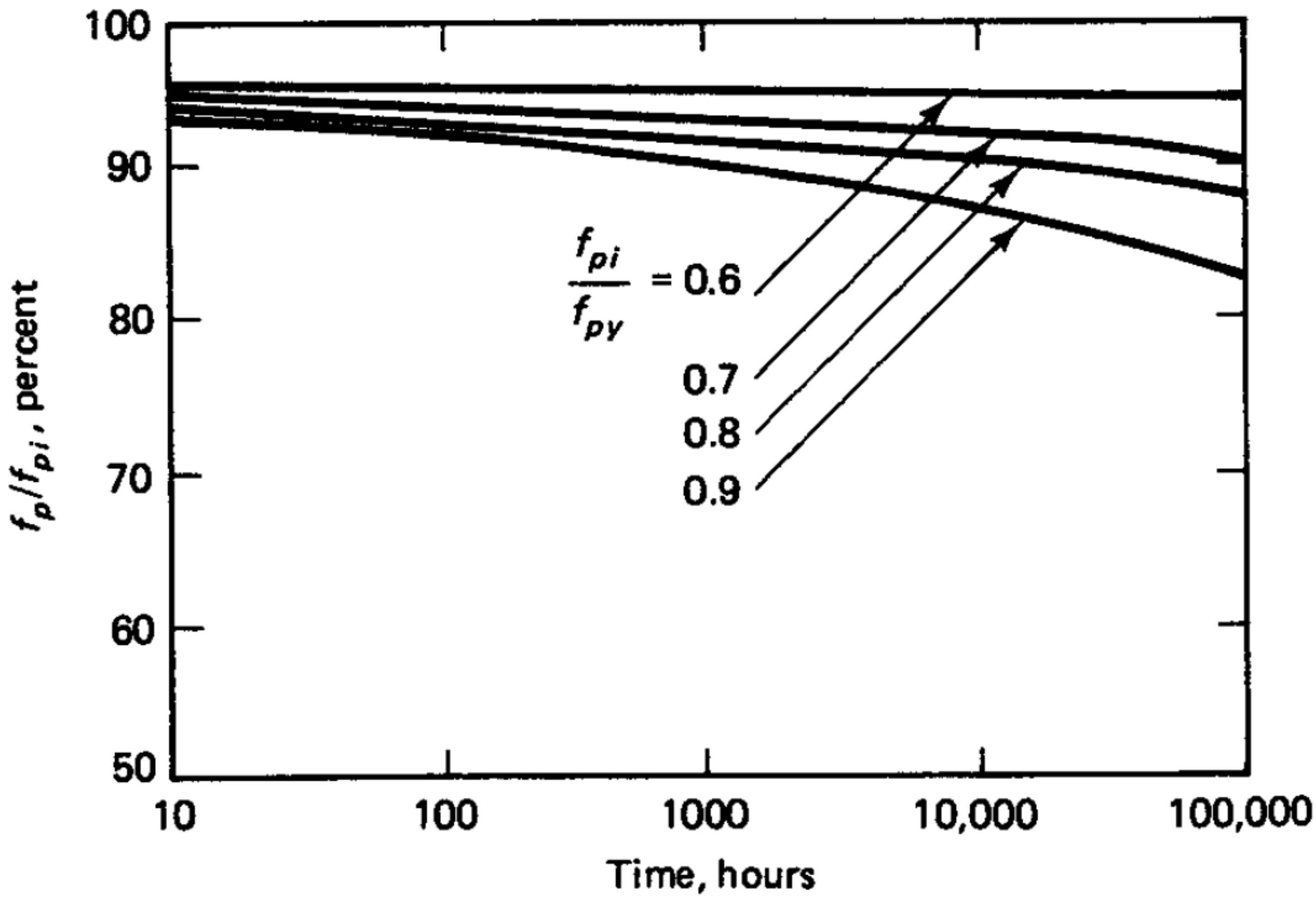
Relaxation of Tendons (RE)



Stress-relaxation relationship in stress-relieved strands.

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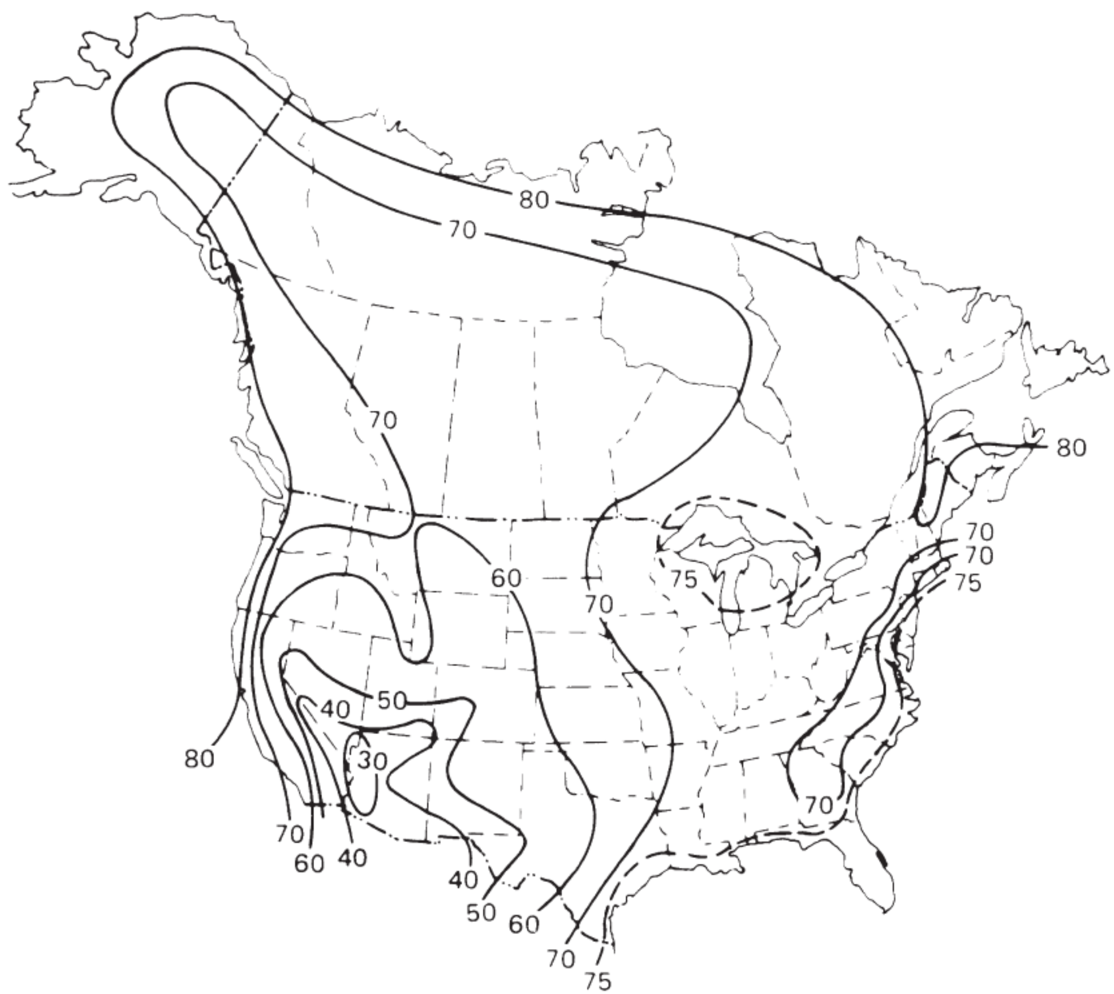
Annual Average Relative Humidity

Annual Average Ambient Relative Humidity (in %)



Approximated Methods for Losses Computations

AASHTO Lump-Sum Losses

AASHTO Lump-Sum Losses

Type of prestressing steel	Total loss $f'_c = 4{,}000$ psi (27.6 N/mm^2)	Total loss $f'_c = 5{,}000$ psi (34.5 N/mm^2)
Pretensioning strand		45,000 psi (310 N/mm^2)
Posttensioning[a] wire or strand	32,000 psi (221 N/mm^2)	33,000 psi (228 N/mm^2)
Bars	22,000 psi (152 N/mm^2)	23,000 psi (159 N/mm^2)

[a] Losses due to friction are excluded. Such losses should be computed according to Section 6.5 of the AASHTO specifications.

Approximate Prestress Loss Values for Posttensioning

Posttensioning tendon material	Prestress loss, psi: Slabs	Prestress loss, psi: Beams and joists
Stress-relieved 270-K strand and stress-relieved 240-K wire	30,000 (207 N/mm^2)	35,000 (241 N/mm^2)
Bar	20,000 (138 N/mm^2)	25,000 (172 N/mm^2)
Low-relaxation 270-K strand	15,000 (103 N/mm^2)	20,000 (138 N/mm^2)

Types of Prestress Losses

Type of prestress loss	Stage of occurrence: Pretensioned members	Stage of occurrence: Posttensioned members	Tendon stress loss: During time interval (t_i, t_j)	Tendon stress loss: Total or during life
Elastic shortening of concrete (ES)	At transfer	At sequential jacking	...	Δf_{pES}
Relaxation of tendons (R)	Before and after transfer	After transfer	$\Delta f_{pR}\ (t_i, t_j)$	Δf_{pR}
Creep of concrete (CR)	After transfer	After transfer	$\Delta f_{pCR}\ (t_i, t_j)$	Δf_{pCR}
Shrinkage of concrete (SH)	After transfer	After transfer	$\Delta f_{pSH}\ (t_i, t_j)$	Δf_{pSH}
Friction (F)	...	At jacking	...	Δf_{pF}
Anchorage seating loss (A)	...	At transfer	...	Δf_{pA}
Total	Life	Life	$\Delta f_{pT}\ (t_i, t_j)$	Δf_{pT}

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Pre-stressed Losses

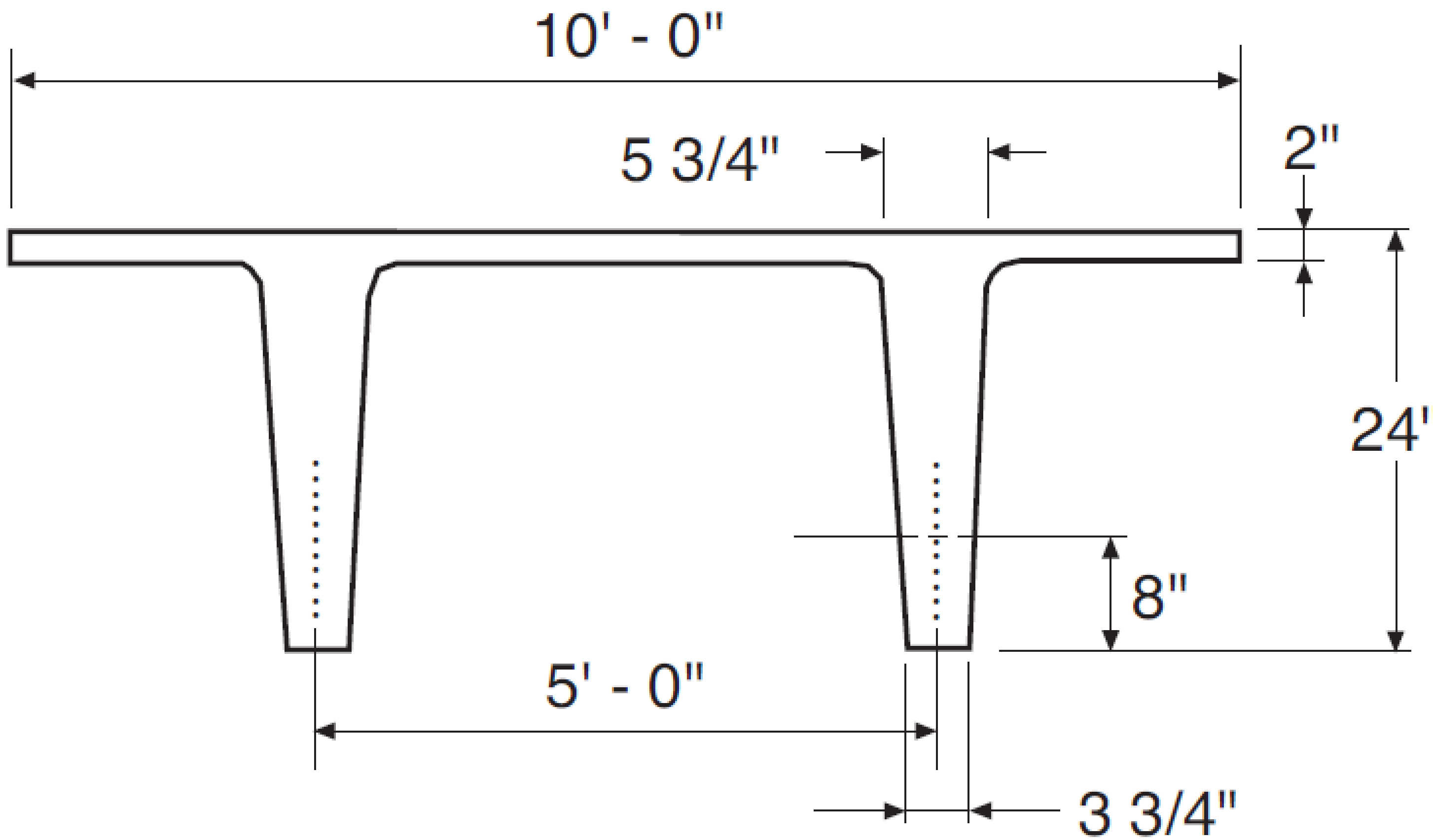
Girder	Measured[1]	PCI-BDM		AASHTO-LRFD Specifications			
				Refined		Lump-sum	
	Loss	Loss	Ratio*	Loss	Ratio*	Loss	Ratio*
Nebraska G1	31.96	36.85	1.15	52.24	1.63	50.29	1.57
Nebraska G2	35.65	38.27	1.07	52.24	1.47	50.29	1.41
New Hampshire G3	43.51	39.84	0.92	54.26	1.25	50.51	1.16
New Hampshire G4	42.33	39.84	0.94	54.26	1.28	50.51	1.19
Texas G7	25.35	32.11	1.27	52.52	2.07	48.83	1.93
Washington G18	42.06	40.33	0.96	66.86	1.59	52.69	1.25
Washington G19	39.98	40.33	1.01	66.86	1.67	52.69	1.32
Average	---	---	1.05	---	1.57	---	1.41
Standard deviation	---	---	0.12	---	0.26	---	0.25

[1] Modified for time infinity.

* Ratio to measured losses.

EXAMPLE

For the simply supported double-tee shown below, estimate loss of prestress using the procedures as outlined earlier under "Computation of Losses." Assume the unit is manufactured in Green Bay, WI.



EXAMPLE

live load = 40 psf

roof load = 20 psf

dead load = 47 psf = 468 plf

span = 48 ft

$f'_{ci} = 3500$ psi

$f'_c = 5000$ psi

8 - 0.5 in. diameter low-relaxation strands

$A_{ps} = 8\,(0.153 \text{ in.}^2) = 1.224 \text{ in.}^2$

e = 9.77 in. (all strands straight)

$f_{pu} = 270{,}000$ psi

$f_{py} = 0.90 f_{pu}$

jacking stress $= 0.74 f_{pu} = 200$ ksi

EXAMPLE

Assume the following for loss computations:

$E_{ci} = 3590$ ksi

$E_c = 4290$ ksi

$E_s = 28{,}500$ ksi

<u>Section Properties</u>

$A_c = 449 \text{ in.}^2$

$I_c = 22{,}469 \text{ in.}^4$

$y_b = 17.77$ in.

$y_t = 6.23$ in.

$V/S = 1.35$ in.

1. Elastic Shortening of Concrete (ES); using Eq. (1)

$$ES = K_{es}E_s \frac{f_{cir}}{E_{ci}} = 1.0\,(28{,}500)\,\frac{0.725}{3590} = 5.8 \text{ ksi}$$

where

$$K_{es} = 1.0 \text{ for pretensioned members}$$

$$f_{cir} = K_{cir}f_{cpi} - f_g$$

$$= K_{cir}\left(\frac{P_{pi}}{A_c} + \frac{P_{pi}e^2}{I_c}\right) - \frac{M_d e}{I_c}$$

$$= 0.9\left(\frac{245}{449} + \frac{245 \times 9.77^2}{22{,}469}\right) - \frac{1617 \times 9.77}{22{,}469} = 0.725 \text{ ksi}$$

$$K_{cir} = 0.9 \text{ for pretensioned members}$$

EXAMPLE

$$P_{pi} = 0.74 f_{pu} A_{ps} = 0.74\,(270)\,(1.224) = 245 \text{ kips}$$

$$M_d = 0.468 \times 48^2 \times \frac{12}{8} = 1617 \text{ in.-kips (dead load of unit)}$$

2. Creep of Concrete (CR); using Eq. (3)

$$CR = K_{cr}\frac{E_s}{E_c}(f_{cir} - f_{cds}) = 2.0 \times \frac{28,500}{4290}(0.725 - 0.30) = 5.6 \text{ ksi}$$

$$\text{where } f_{cds} = M_{ds}\frac{e}{I_c} = 691 \times \frac{9.77}{22,469} = 0.30 \text{ ksi}$$

$$M_{ds} = 0.02 \times 10 \times 48^2 \times \frac{12}{8} = 691 \text{ in.-kips (roof load only)}$$

and $K_{cr} = 2.0$ for pretensioned members.

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EXAMPLE

3. Shrinkage of Concrete (SH); using Eq. (4)

$$SH = 8.2 \times 10^{-6} K_{sh} E_s \left(1 - 0.06 \frac{V}{S}\right) (100 - RH)$$

$$= 8.2 \times 10^{-6} \times 1.0 \times 28{,}500\ (1 - 0.06 \times 1.35)\ (100 - 75) = 5.4 \text{ ksi}$$

RH = average relative humidity surrounding the concrete member from Fig.
For Green Bay, Wisconsin, RH = 75%

and K_{sh} = 1.0 for pretensioned members.

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EXAMPLE

4. Relaxation of Tendon Stress (RE); using Eq. (5)

$$RE = [K_{re} - J\,(SH + CR + ES)]\,C$$

$$= [5 - 0.04\,(5.4 + 5.6 + 5.8)]\,0.95 = 4.1 \text{ ksi}$$

where, for 270 Grade low-relaxation strand:

K_{re} = 5 ksi Table 2

J = 0.040 Table 2

C = 0.95 Table 3 for $\frac{f_{pi}}{f_{pu}}$ = 0.74)

5. Total allowance for loss of prestress

$$ES + CR + SH + RE = 5.8 + 5.6 + 5.4 + 4.1 = 20.9 \text{ ksi}$$

6. Stress, f_p, and force, P_p, immediately after transfer.

Assume that one-fourth of relaxation loss occurs prior to release.

$$f_p = 0.74 f_{pu} - (ES + 1/4\ RE)$$

$$= 0.74\,(270) - [5.8 + 1/4\,(4.1)] = 193.0 \text{ ksi}$$

$$P_p = f_p A_{ps} = 193.0 \times 1.224 = 236 \text{ kips}$$

7. Effective prestress stress f_{se} and effective prestress force P_e after all losses

$$f_{se} = 0.74 f_{pu} \text{ - allowance for all prestress losses}$$

$$= 0.74\,(270) - 20.9 = 179 \text{ ksi}$$

$$P_e = f_{se} A_{ps} = 179 \times 1.224 = 219 \text{ kips}$$

ABC: Vigas Pretensadas para Puentes Principios Básicos de Diseño

Pre-stressed concrete Design

Service Design

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Introduction to the Service Design Concepts

Serviceability Limit State Design (SLS) *is design of prestressed concrete member at the service load stage. The SLS design is related with the* ***working stress design*** *method because the* ***allowable stress limit*** *controls the design process. SLS design approach, control the selection of the* ***geometrical dimension*** *and the* ***layout of the prestressing steel*** *regardless if it is pre-tensioned or post-tensioned member. After the SLS design is satisfied then the other factor such as* ***shear design****,* ***torsion design****,* ***ultimate strength design****,* ***control of deflection and cracking*** *are satisfied. Different with reinforced concrete design, in prestressed concrete design several load stage must be checked such as when* ***load transfer stage****,* ***service load stage*** *and* ***ultimate load stage****.*

Basic Assumptions in the Service Design Method

There are three basic concepts may be applied to design the prestressed concrete member using, as follows:

1. The material is assumed as an ***elastic composite Material***.

2. The ***compressive stress is carried by the concrete material*** and the ***tensile stress is carried*** by ***the prestressing steel***.

3. Pre-stressed concrete section is assumed ***un-cracked due to service load***

Loading Stages

Pre-stressed concrete design must consider all the loading stages started from the ***initial pre-stressing force*** until the ***limit state of failure***. For each loading stage the ***actual stress must be checked*** and **compare *with the allowable stress*** defined by the code. At the ultimate stage the flexural capacity **must** be compared with the ultimate bending moment or shear. For serviceability limit state design the ***two loading stages may governs the design***, as follows :

1. *Initial Loading* , a loading stage where the ***prestressing force is transferred to the concrete*** without any external loading except the self weight of the structural member.

2. *Final Service Loading*, a loading stage where the ***prestressing force already reduced by several losses*** and full service load is applied.

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Loading Stages

Complete loading stages for ***Serviceability limit state design*** of prestressed concrete member are as follows :

Initial Loading

a. *Initial Prestressing Force*.

b. *Self Weight (DL)*

Final Service Loading

a. *Full Dead Load (SDL) + Effective Prestressing Force*.

b. *Full Service Load (DL+SDL+LL) + Effective Prestressing Force*.

The loading stages for of prestressed concrete member at **Ultimate limit state design** are as follows :

Full Ultimate Load (α *DL* + β *SDL* + δ *LL*)

Where: α β δ *are combination Factors*

Initial Loading Stage

THE TRANSFER STAGE

The intial pre-stresing force P_i is acting in the concrete element, after the ***Transfer stage*** occurs. The process of transferring the force acting in the hydraulic jack to the concrete element is called "The transfer stage" The external load acting at this stage is normally the ***self weight (DL)*** of the structural member. After the force acting in the hydraulic jack is transferred, the concrete element is ***under one of several critical*** conditions for the reasons described as it follow follows :

1. The ***pre-stressing force is maximum*** because the losses has not yet occurred.
2. The acting external loading is ***minimum***.
3. The ***concrete strength is minimum*** as the concrete is still young.

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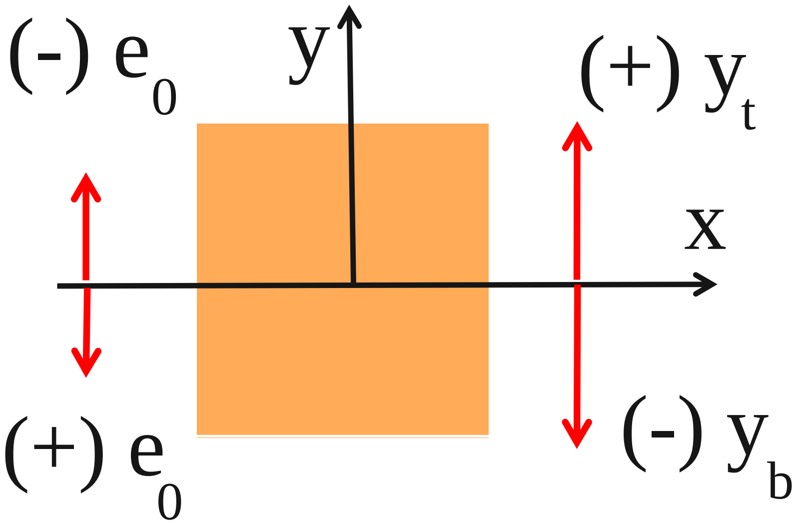
Service Loading Stage

Service load stage is a loading stage where the ***pre-stress long term loss*** already taken. The pre-stress long term loss already taken is the creep loss, shrinkage loss and steel relaxation loss. The external loading at this stage is the ***full service load*** these are ***self weight (DL), superimposed dead load (SDL)*** and ***live load (LL)***.

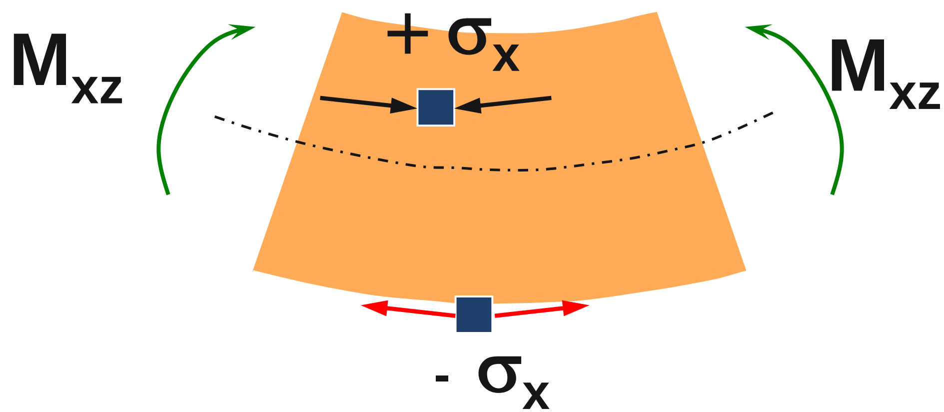
This stage is ***critical*** because of as follows :

1. The ***pre stressing force is minimum*** because all the losses already taken.
2. The applied loading is ***maximum*** because all the service loads already applied.
3. The pre stressing force at this loading stage is designated as ***Pe***.

Stresses and Moments Sign Convention



M_{xz}=Bending Moment



$$\sigma_{top}(+) = \frac{(+)M(+)y_t}{I}$$

$$\sigma_{bot}(-) = \frac{(+)M(-)y_t}{I}$$

"Sad" Beam is (-)

"Happy" Beam is (+)

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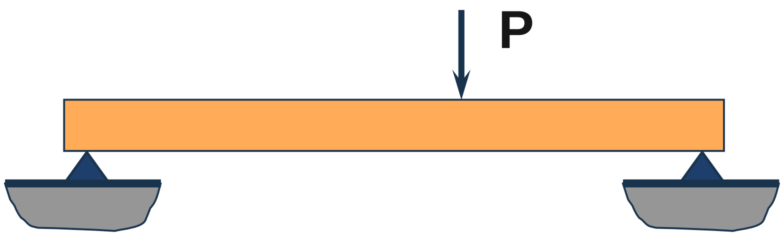
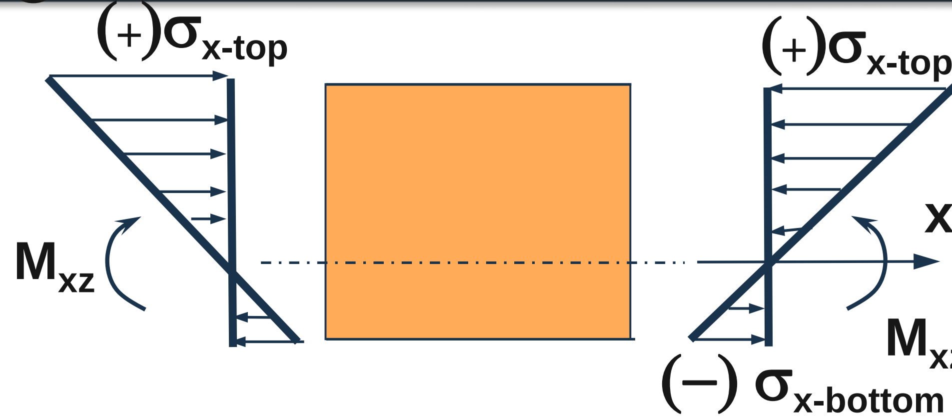
Stresses Sign Convention

Stresses in Beams

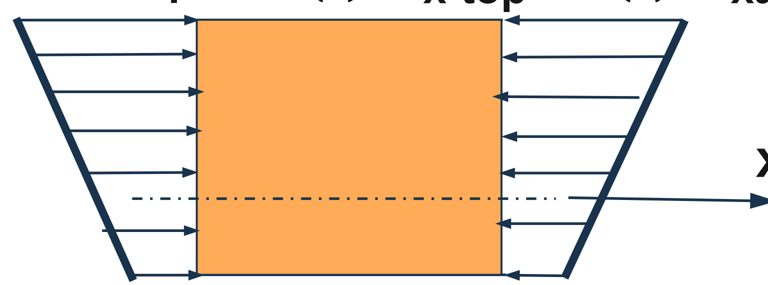




Axial Load in Beams

$$\sigma_{top} = (+)\,\sigma_{x\text{-top}} + (+)\,\sigma_{xa}$$

Combined Bending and Axial Loading

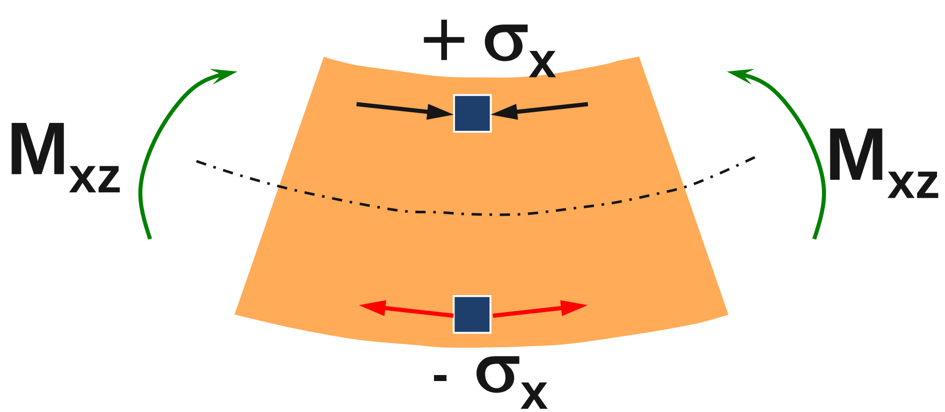


$$\sigma_{bottom} = (-)\,\sigma_{x\text{-bottom}} + (+)\,\sigma_{xa}$$

What's Look Different ????

There is nothing new, however to be consistent with many authors, now Compression stresses are positive and tension stresses are negative

M_{xz}=Bending Moment



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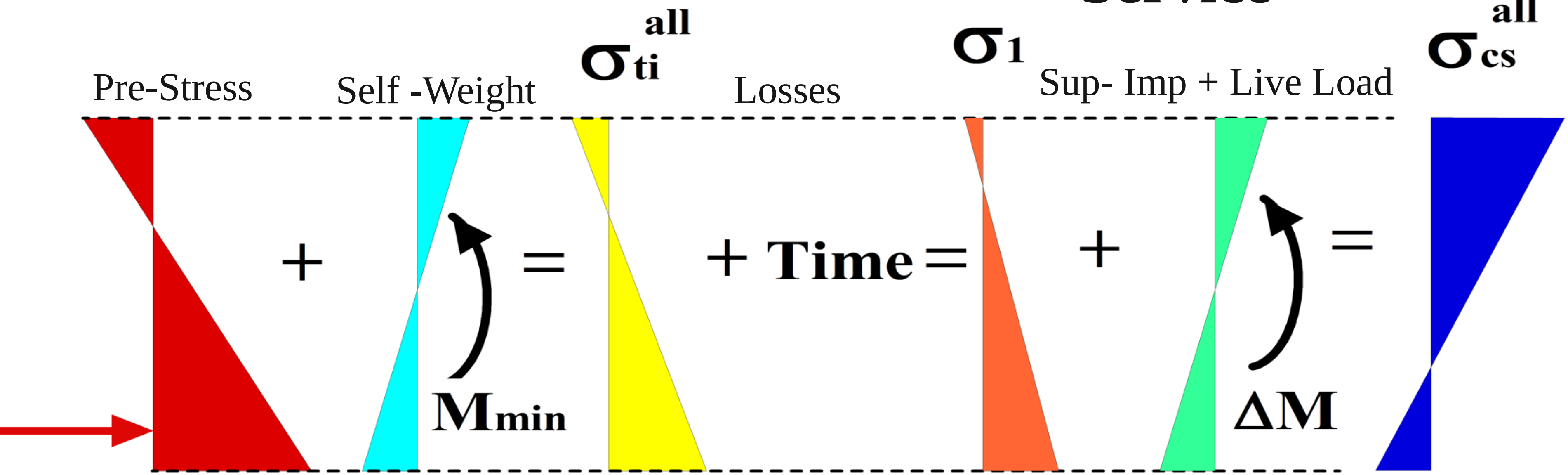
Stress Diagram

The following figure shows the stress diagram for each loading stage, as follows :

Initial **Service**



$$F_i + M_{min} = \sigma_{ci}^{all} + \text{Time} = \sigma_2 + \Delta M = \sigma_{ts}^{all}$$

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Loading- Stresses

Initial Loading – Stress Expression

$$\sigma^{i}_{top} = \frac{P_i}{A} - \frac{P_i(e_0)y_t}{I} + \frac{M_{DL}y_t}{I} = \sigma_{ti}$$

$$\sigma^{i}_{bot} = \frac{P_i}{A} + \frac{P_i(e_0)y_b}{I} - \frac{M_{DL}y_b}{I} = \sigma_{ci}$$

Final Service Loading - Stress Expression

$$\sigma^{s}_{top} = \frac{P_e}{A} - \frac{P_e(e_0)y_t}{I} + \frac{M_{DL}y_t}{I} + \frac{M_{SDL}y_t}{I} + \frac{M_{LL}y_t}{I} = \sigma_{cs}$$

$$\sigma^{s}_{bot} = \frac{P_e}{A} + \frac{P_e(e_0)y_b}{I} - \frac{M_{DL}y_b}{I} - \frac{M_{SDL}y_b}{I} - \frac{M_{LL}y_b}{I} = \sigma_{ts}$$

σ_{ci} = Initial concrete compressive ***(transfer stage)***

σ_{ti} = Initial concrete tensile stress ***(transfer stage)***

σ_{cs} = Service concrete compressive stress ***(service load stage)***

σ_{ts} = Service concrete tensile stress ***(service load stage)***

M_{DL} = Moment due to the beam self weight

M_{SDL} = Moment due to superimposed dead load

M_{LL} = Moment due to acting live load

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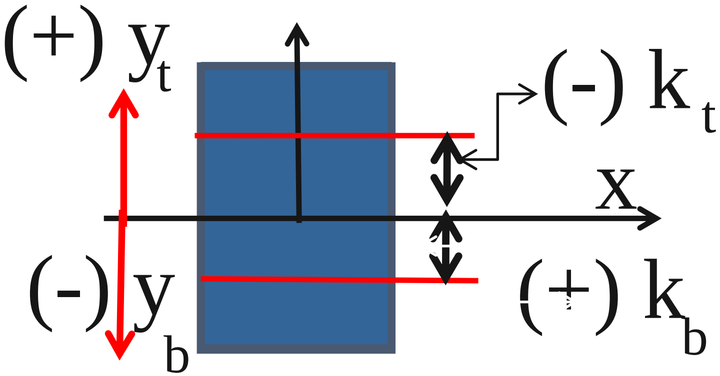
Kern Center Concepts

The Max eccentricity limit so that no tensile stress will develop any where in the member could be found by setting $\sigma = 0$; $0 = \frac{N}{A} +/- \frac{(Ne)y}{I}$

or: $\frac{1}{A}(1 +/- \frac{(e)y}{r^2}) = 0$

or: $e_t = k_t = (-)\frac{S_b}{y_b}$; $e_b = k_b = (+)\frac{S_b}{y_t}$



ALLOWABLE SERVICE STRESSES DENOMINATION

σ_{ci}^{all} = concrete compressive stress ***(transfer stage)***
σ_{ti}^{all} = concrete tensile stress ***(transfer stage)***
σ_{cs}^{all} = concrete compressive stress ***(service load stage)***
σ_{ts}^{all} = concrete tensile stress ***(service load stage)***

$M_{min} = M_{DL}$; $M_{max} = M_{DL} + M_{SDL} + M_{LL}$; $P_i = F_i$; $P_e = F_e = \eta \times F_i$

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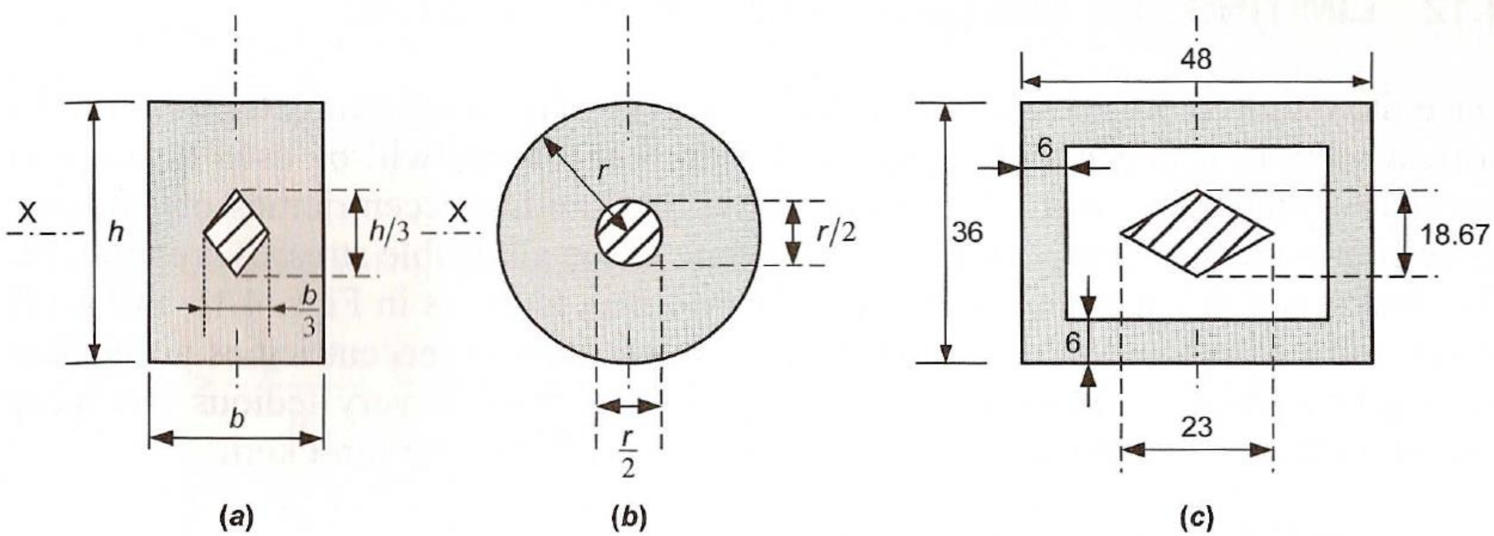
Central Kern

Central kern is a region which is an axial compressive force of any magnitude will not produce any tension at this section.

The central kern is depends to the cross section. The central kern is independent to the applied compressive force and allowable stre

$$\text{Typical equations for xx axis: } k_t = -\frac{Z_b}{A_c} = -\frac{I}{A_c y_b}; \qquad k_b = \frac{Z_t}{A_c} = \frac{I}{A_c y_t}$$



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Basic Concepts

Initial Loading – Stress Expression

$$\sigma^{i}_{top} = \frac{P_i}{A} - \frac{P_i (e_0) y_t}{I} + \frac{M_{DL} y_t}{I} = \sigma_{ti}$$

$$\sigma^{i}_{bot} = \frac{P_i}{A} + \frac{P_i (e_0) y_b}{I} - \frac{M_{DL} y_b}{I} = \sigma_{ci}$$

Geometry Related

Code Related

Geometry Limits

Final Service Loading - Stress Expression

$$\sigma^{s}_{top} = \frac{P_e}{A} - \frac{P_e (e_0) y_t}{I} + \frac{M_{DL} y_t}{I} + \frac{M_{SDL} y_t}{I} + \frac{M_{LL} y_t}{I} = \sigma_{cs}$$

$$\sigma^{s}_{bot} = \frac{P_e}{A} + \frac{P_e (e_0) y_b}{I} - \frac{M_{DL} y_b}{I} - \frac{M_{SDL} y_b}{I} - \frac{M_{LL} y_b}{I} = \sigma_{ts}$$

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Pre-stressed Equations to Remember

Initial Loading – Stress Expression

$$\sigma^{i}_{top} = \frac{P_i}{A} - \frac{P_i(e_0)y_t}{I} + \frac{M_{DL}\,y_t}{I} = \sigma_{ti} = \frac{P_i}{A} \times \left[1 - \left(\frac{e_0}{k_b}\right)\right] + \frac{M_{DL}}{S_t} \geq \sigma_{ti}^{Allow}$$

Be Careful !!!!!

$$\sigma^{i}_{bot} = \frac{P_i}{A} + \frac{P_i(e_0)y_b}{I} - \frac{M_{DL}\,y_b}{I} = \sigma_{ci} = \frac{P_i}{A} \times \left[1 + \left(\frac{e_0}{(-)k_t}\right)\right] - \frac{M_{DL}}{S_t} \leq \sigma_{ci}^{Allow}$$

Sign Convention Summary

The sign convention for bending moment is as follows :

1. *Plus sign (+)* is used to for ***positive bending moment*** which is ***compression in the top fiber*** and ***tension in the bottom fiber***.

2. *Minus sign (-)* is used to for ***negative bending moment*** which is ***tension in the top fiber*** and ***compression in the bottom fiber***.

The sign convention for bending moment due to pre-stressing eccentricity is as follows :

1. The bending moment is ***positive (+)*** if the eccentricity goes ***up ward / above the neutral axis***.

2. The bending moment is ***negative (-)*** if the eccentricity goes ***down ward / below the neutral axis***.

The sign convention for pre stressing force is as follows :

1. The pre-stressing force is always in ***plus (+) sign***.

The sign convention for pre-stressing eccentricity is as follows :

1. The eccentricity is ***positive (+)*** if goes ***down ward / below the neutral axis***.

2. The eccentricity is ***negative (-) if*** goes ***up ward / above the neutral axis***.

The sign convention for concrete stress is as follows :

1. Plus sign ***(+)*** is used to for ***compressive concrete stress***.

2. Minus sign ***(-)*** is used for ***tensile concrete stress***.

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Loading Stages - Stresses

Initial Loading – Stress Expression

$$\sigma^{i}_{top} = \frac{P_i}{A} - \frac{P_i(e_0)y_t}{I} + \frac{M_{DL}y_t}{I} = \sigma_{ti} = \frac{P_i}{A}\times[1-(\frac{e_0}{k_b})] + \frac{M_{DL}}{S_t} \geq \sigma_{ti}^{Allow}$$

$$\sigma^{i}_{bot} = \frac{P_i}{A} + \frac{P_i(e_0)y_b}{I} - \frac{M_{DL}y_b}{I} = \sigma_{ci} = \frac{P_i}{A}\times[1+(\frac{e_0}{(-)k_t})] - \frac{M_{DL}}{S_t} \leq \sigma_{ci}^{Allow}$$

Final Service Loading - Stress Expression

$$\sigma^{S}_{top} = \frac{P_e}{A} - \frac{P_e(e_0)y_t}{I} + \frac{M_{DL}y_t}{I} + \frac{M_{SDL}y_t}{I} + \frac{M_{LL}y_t}{I} = \sigma_{cs}$$

$$\sigma^{S}_{top} = \sigma_{cs} = \frac{P_e}{A}\times[1-(\frac{e_0}{k_b})] + \frac{M_{DL}}{S_t} + \frac{M_{SDL}}{S_t} + \frac{M_{LL}}{S_t} \leq \sigma_{cs}^{Allow}$$

$$\sigma^{S}_{bot} = \frac{P_e}{A} + \frac{P_e(e_0)y_b}{I} - \frac{M_{DL}y_b}{I} - \frac{M_{SDL}y_b}{I} - \frac{M_{LL}y_b}{I} = \sigma_{ts}$$

$$\sigma^{S}_{bot} = \sigma_{ts} = \frac{P_e}{A}\times[1+(\frac{e_0}{(-)k_t})] - \frac{M_{DL}}{S_b} - \frac{M_{SDL}}{S_b} - \frac{M_{LL}}{S_b} \geq \sigma_{ts}^{Allow}$$

Using the the previous equations will be re-writted as:

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Loading Stages - Stresses

Initial Loading – Stress Expression

$$\sigma^{i}_{top} = \sigma_{ti} = \frac{F_i}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{min}}{S_t} \geq \sigma_{ti}^{Allow}$$

$$\sigma^{i}_{bot} = \sigma_{ci} = \frac{F_i}{A} \times [1 + (\frac{e_0}{(-)k_t})] - \frac{M_{min}}{S_t} \leq \sigma_{ci}^{Allow}$$

Final Service Loading - Stress Expression

$$\sigma^{s}_{top} = \sigma_{cs} = \frac{\eta \times F_i}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{max}}{S_t} \leq \sigma cs^{Allow}$$

$$\sigma^{s}_{bot} = \sigma_{ts} = \frac{\eta \times F_i}{A} \times [1 + (\frac{e_0}{(-)k_t})] - \frac{M_{max}}{S_b} \geq \sigma_{ts}^{Allow}$$

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Loading Stages- Exentricity

Initial Loading – Excentricity

$$e_0 \le k_b + \left(\frac{1}{F_i}\right) \times \left(M_{min} - \sigma_{ti}^{Allow} \times S_t\right)$$

$$e_0 \le k_t + \left(\frac{1}{F_i}\right) \times \left(M_{min} + \sigma_{ci}^{Allow} \times S_b\right)$$

Final Service Loading - Excentricity Expression

$$e_0 \ge k_b + \left(\frac{1}{\eta F_i}\right) \times \left(M_{max} - \sigma_{cs}^{Allow} \times S_t\right)$$

$$e_0 \ge k_t + \left(\frac{1}{\eta F_i}\right) \times \left(M_{max} + \sigma_{ts}^{Allow} \times S_b\right)$$

Loading Stages- Forces

Initial Loading – Force Expression

$$F_i = (M_{min} - \sigma_{ti}^{Allow} \times S_t)/(e_0 - k_b)$$

$$F_i = (M_{min} + \sigma_{ci}^{Allow} \times S_b)/(e_0 - k_t)$$

Final Service Loading - Force Expression

$$\eta F_i = (M_{max} - \sigma_{cs}^{Allow} \times S_t)/(e_0 - k_b)$$

$$\eta F_i = (M_{max} + \sigma_{ts}^{Allow} \times S_b)/(e_0 - k_t)$$

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Loading Stages - Forces

Initial Loading – Force Expression

$$\frac{1}{F_i} = (e_0 - k_b)/(M_{min} - \sigma_{ti}^{Allow} \times S_t)$$

$$\frac{1}{F_i} = (e_0 - k_t)/(M_{min} + \sigma_{ci}^{Allow} \times S_b)$$

Final Service Loading - Force Expression

$$\frac{1}{\eta F_i} = (e_0 - k_b)/(M_{max} - \sigma_{cs}^{Allow} \times S_t)$$

$$\frac{1}{\eta F_i} = (e_0 - k_t)/(M_{max} + \sigma_{ts}^{Allow} \times S_b)$$

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Service Condition Allowable Stresses

As explained before that pre-stressed concrete design in the serviceability limit state is controlled by the allowable stress determined by the code. The allowable stress is both for ***allowable stress in concrete*** and ***allowable stress in pre-stressing steel***. There are at least four stress limitations, as follows :

1. Tensile stress and compressive stress at ***transfer stage***.
2. Tensile stress and compressive stress at ***service load stage***.

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Service Condition Allowable Stresses

Allowable concrete stresses for transfer stage and service load stage

STRESS	TRANSFER STAGE		SERVICE LOAD STAGE	
	Psi	MPa	Psi	MPa
Compressive	$\bar{f}_{ci} = 0.60f'_{ci}$	$\bar{f}_{ci} = 0.60f'_{ci}$	$\bar{f}_c = 0.45f'_c$	$\bar{f}_c = 0.45f'_c$
Tensile	$\bar{f}_{ti} = 3\sqrt{f'_{ci}}$	$\bar{f}_{ti} = \frac{1}{4}\sqrt{f'_{ci}}$	$\bar{f}_t = 6\sqrt{f'_c}$	$\bar{f}_t = \frac{1}{2}\sqrt{f'_c}$
Tensile (At support of simple beam)	$\bar{f}_{ti} = 6\sqrt{f'_{ci}}$	$\bar{f}_{ti} = \frac{1}{2}\sqrt{f'_{ci}}$	$\bar{f}_t = 6\sqrt{f'_c}$	$\bar{f}_t = \frac{1}{2}\sqrt{f'_c}$
Tensile (One way system, long term deflection considered)	$\bar{f}_{ti} = 3\sqrt{f'_{ci}}$	$\bar{f}_{ti} = \frac{1}{4}\sqrt{f'_{ci}}$	$\bar{f}_t = 12\sqrt{f'_c}$	$\bar{f}_t = \sqrt{f'_c}$

f_{ci} = allowable concrete compressive stress ***(transfer stage)***
f_{ti} = allowable concrete tensile stress ***(transfer stage)***
f_c = allowable concrete compressive stress ***(service load stage)***
f_t = allowable concrete tensile stress ***(service load stage)***
f_{ci} = initial concrete compressive strength
f'_c = 28 days concrete compressive strength

Service Condition Allowable Stresses

Allowable steel stresses for transfer stage and service load stage

STRESS	JACKING	TRANSFER STAGE	
		PRE-TENSIONED AFTER PRESTRESS TRANSFER	POST-TENSIONED AFTER TENDON ANCHORAGE
Tensile	$f_{pJ} = 0.94f_{py} \leq 0.80f_{pu}$	$f_{pi} = 0.82f_{py} \leq 0.74f_{pu}$	$f_{pJ} = 0.80f_{pu}$

where :

f_{pJ} = jacking stress

f_{pi} = initial tendon stress

f_{py} = yield strength of prestressing steel

f_{pu} = ultimate tensile strength of prestressing steel

The value of f_{py} is as follows :

TABLE 7.4 YIELD STRENGTH OF PRESTRESSING STEEL

TYPE	YIELD STRENGTH
Stress Relieved Tendon	$f_{py} = 0.85f_{pu}$
Low Relaxation Tendon	$f_{py} = 0.90f_{pu}$
Prestressing Bar	$f_{py} = 0.80f_{pu}$

Design of Flexural Members

The design process is more difficult than analysis process because the unknown variables are more than when the section is analyzed. In the design process When need to find the minimum section to satisfy the allowable stress. After the section is determined then the actual stress also must be checked. Two governs condition are the maximum pres-tressing force with minimum external load and minimum pre-stressing force with maximum external load.

ECCENTRIC PRESTRESSING

The figure below shows the stress history of the eccentric pre-stressing system. Two stress conditions are used to determine the minimum section modulus. For eccentric pre-stressing the minimum section is controlled by maximum eccentricity at mid span.

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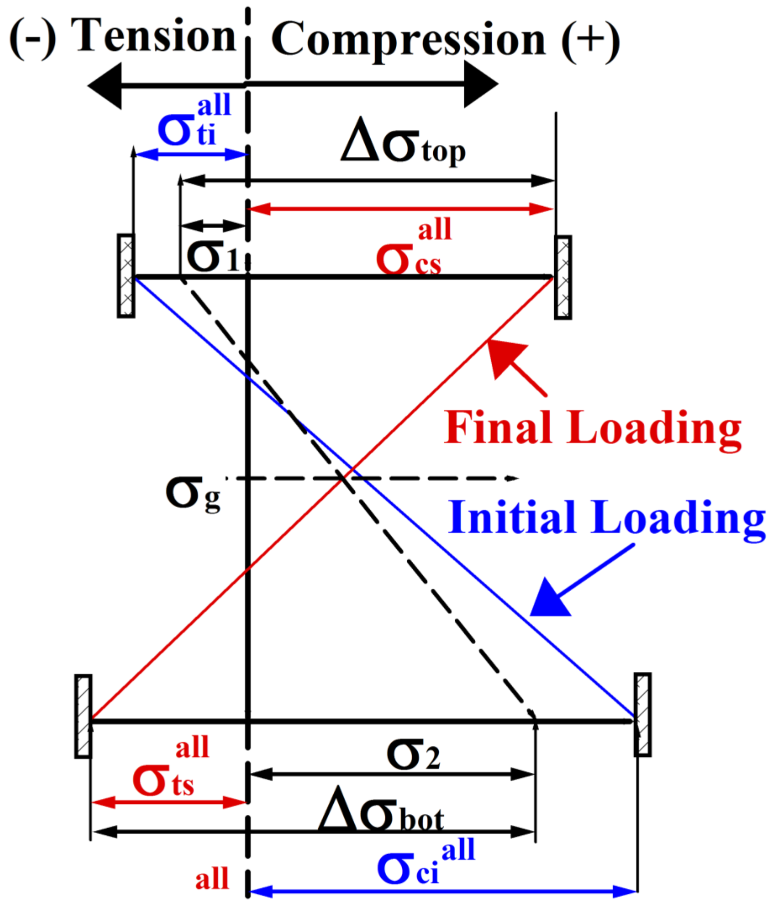
Stress History



$\sigma_{cs}^{all} > \sigma_1 + \Delta\sigma_{top}$

$\sigma_2 + \Delta\sigma_{bot} > \sigma_{ts}$

Stress History

The followings are the explanation of the stress history, as follows :

The actual stress due to pre-stressing and external load is shown with dashed line. Due to initial pre-stressing the stress will be tension in the top fiber (must be less than allowable tensile stress fti) and compression in the bottom fiber (must be less than allowable compressive stress fci).Due to effective pre-stressing the stress will be compression in the top fiber (must be less than allowable compressive stress fc) and tension in the bottom fiber (must be less than allowable tensile stress ft).The top fiber is controlled by the initial allowable tensile stress fti and allowable compressive stress fc. The bottom fiber is controlled by the initial allowable compressive stress fci and allowable tensile stress f_t .

PRE-STRESSED CONCRETE SEMINAR

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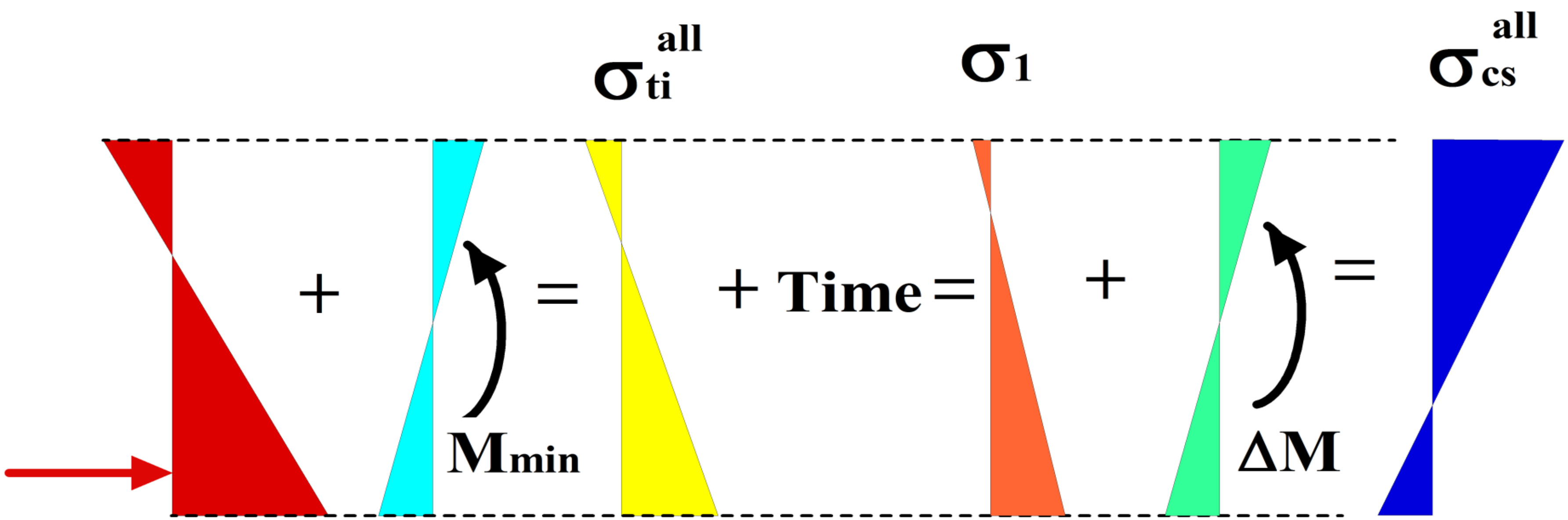
Stress History



$$F_i + M_{min} = \sigma_{ci}^{all} + \text{Time} = \sigma_2 + \Delta M = \sigma_{ts}^{all}$$

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Stress History

As previously explained the top fiber is controlled by the initial allowable tensile stress f_{ti} and allowable compressive stress fc. So the section modulus for the top fiber St will be derived based on the two allowable stresses above.

Initial Loading – Stress Expression

$$\sigma^{i}_{top} = \sigma_{ti} = \frac{P_i}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{min}}{S_t} \geq \sigma_{ti}^{Allow}$$

Final Service Loading - Stress Expression

$$\sigma^{S}_{top} = \sigma_{cs} = \frac{\eta \times F_i}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{max}}{S_t} \leq \sigma cs^{Allow}$$

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Stress History

Assuming that all pre-stressed losses have occurred leads to a loading for which:

$$\frac{\eta \times F_i}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{min}}{S_t} = \sigma 1$$

Which can also be written as :

$$\eta[\frac{\eta \times F_i}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{min}}{S_t}] - \eta \frac{M_{min}}{S_t} - \frac{M_{min}}{S_t} = \sigma 1$$

but: $[\frac{P_i}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{min}}{S_t}] = \sigma_{ti}^{Allow}$

Therefore:

$$\eta \sigma_{ti}^{Allow} + \frac{M_{min}}{S_t}(1 - \eta) = \sigma_1$$

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Stress History

Now if we add a moment of amplitude ΔM to the section the corresponding additional stress on the top fiber will be :

$$\Delta\sigma_{top} = \frac{\Delta M}{S_t}$$

And the resulting stress due to the initial stress at the top plus the additional stress due to the moment ΔM must be less than or equal to the allowable compressive stress

$$\sigma_1 + \Delta\sigma_{top} \leq \sigma cs^{Allow}$$

Using previous equations it results that :

$$\eta\sigma_{ti}^{Allow} + \frac{M_{min}}{S_t}(1-\eta) + \frac{\Delta M}{S_t} \leq \sigma_{cs}^{Allow}$$

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Preliminary Sizing

The previous equation can be expressed as:

$$S_t = \frac{M_{min}(1-\eta) + \Delta M}{\sigma_{cs}^{Allow} - \eta\sigma_{ti}^{Allow}}$$

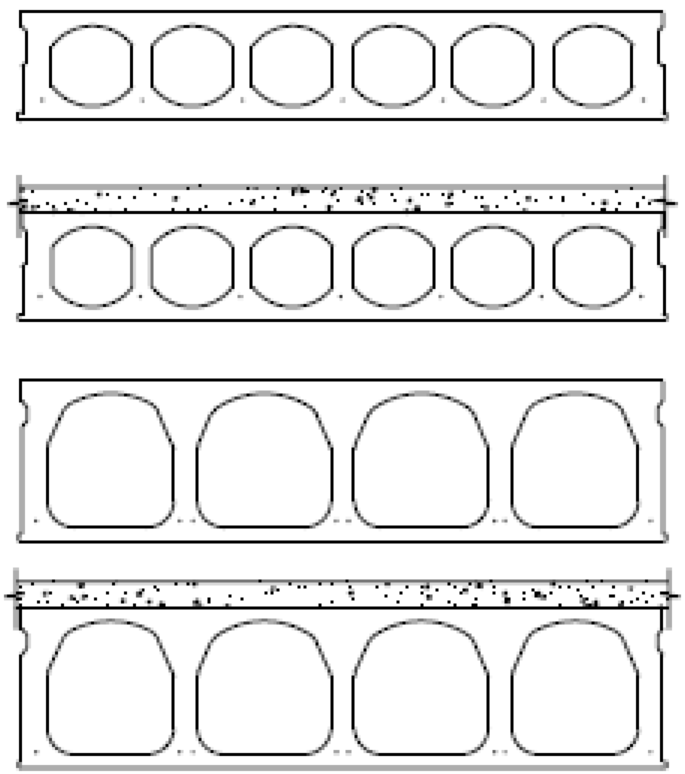
By similarly examining the state of stress on the bottom fibber, it can be shown that:

$$S_b = \frac{M_{min}(1-\eta) + \Delta M}{\eta\sigma_{ci}^{Allow} - \sigma_{ts}^{Allow}}$$

Note that:

$$M_{max} = M_{min} + \Delta M$$

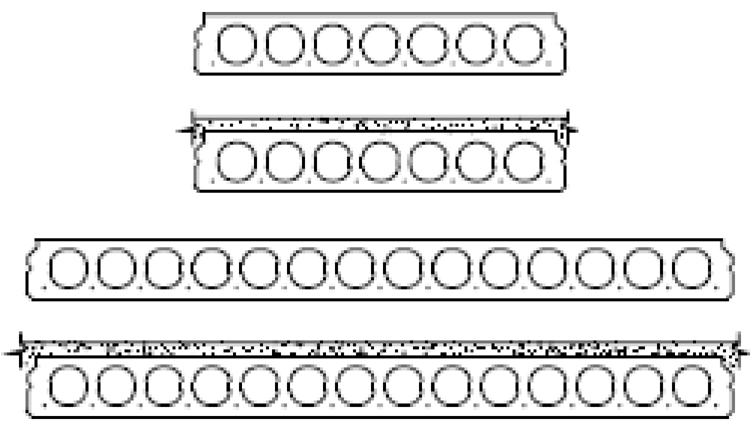
Trade name: Dy-Core
Equipment Manufacturer: Mixer Systems, Inc., Pewaukee, Wisconsin



Section	Untopped				with 2" topping		
width x depth	A in^2	yb in	I in^4	wt psf	yb in	I in^4	wt psf
4'-0" x 6"	142	3.05	661	37	4.45	1475	62
4'-0" x 8"	193	3.97	1581	50	5.43	3017	75
4'-0" x 10"	215	5.40	2783	56	6.89	4614	81
4'-0" x 12"	264	6.37	4773	69	7.89	7313	94
4'-0" x 15"	289	7.37	8604	76	9.21	13225	101

Note: All sections not available from all producers. Check availability with local manufacturers.

Trade name: Dynaspan®
Equipment Manufacturer: Dynamold Corporation, Salina, Kansas



Section	Untopped				with 2" topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
4'-0" x 4"	133	2.00	235	35	3.08	689	60
4'-0" x 6"	165	3.02	706	43	4.25	1543	68
4'-0" x 8"	233	3.93	1731	61	5.16	3205	86
4'-0" x 10"	260	4.91	3145	68	6.26	5314	93
8'-0" x 6"	338	3.05	1445	44	4.26	3106	69
8'-0" x 8"	470	3.96	3525	61	5.17	6444	86
8'-0" x 10"	532	4.96	6422	69	6.28	10712	94
8'-0" x 12"	615	5.95	10505	80	7.32	16507	105

Note: All sections not available from all producers. Check availability with local manufacturers.

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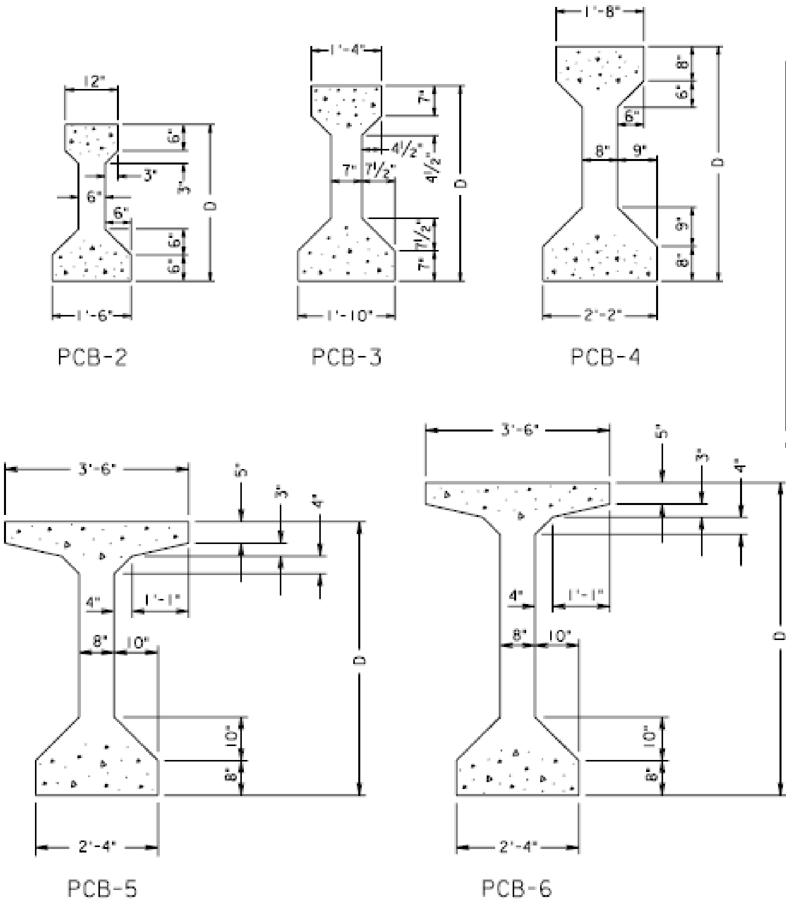

Beam Type	Depth D (in)	Area A (in^2)	Centroid to Bottom y_b (in)	Moment of Inertia I ($x\ 10^3\ in^4$)	Section Modulus S_{top} (in^3)	Section Modulus S_{bott} (in^3)	Weight @ 150 pcf (lbs/lin. ft.)
PCB-2	36	369	15.83	50.98	2528	3220	384
PCB-3	45	560	20.27	125.39	6186	5070	583
PCB-4	54	789	24.73	260.73	8908	10543	822
PCB-5	63	1013	31.96	521.18	16791	16307	1055
PCB-6	72	1085	36.38	733.32	20587	20157	1130

PRESTRESSED CONCRETE
I-BEAMS
SECTION PROPERTIES

VOL. V - PART 2
DATE: 01Jun2005
SHEET 3 of 6
FILE NO. 12.04-3

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Pre-stressed Equations to Remember

Initial Loading – Stress Expression

$$\sigma^{i}_{top} = \frac{P_i}{A} - \frac{P_i(e_0)y_t}{I} + \frac{M_{DL}\,y_t}{I} = \sigma_{ti} = \frac{P_i}{A} \times \left[1 - \left(\frac{e_0}{k_b}\right)\right] + \frac{M_{DL}}{S_t} \geq \sigma_{ti}^{Allow}$$

Be Careful !!!!!

$$\sigma^{i}_{bot} = \frac{P_i}{A} + \frac{P_i(e_0)y_b}{I} - \frac{M_{DL}\,y_b}{I} = \sigma_{ci} = \frac{P_i}{A} \times \left[1 + \left(\frac{e_0}{(-)k_t}\right)\right] - \frac{M_{DL}}{S_t} \leq \sigma_{ci}^{Allow}$$

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Pre-stressed Equations to Remember

Final Service Loading - Stress Expression

$$\sigma^{s}_{top} = \frac{P_e}{A} - \frac{P_e(e_0)y_t}{I} + \frac{M_{DL}\,y_t}{I} + \frac{M_{SDL}\,y_t}{I} + \frac{M_{LL}\,y_t}{I} = \sigma_{cs}$$

$$\sigma^{s}_{top} = \sigma_{cs} = \frac{P_e}{A} \times [1 - (\frac{e_0}{k_b})] + \frac{M_{DL}}{S_t} + \frac{M_{SDL}}{S_t} + \frac{M_{LL}}{S_t} \leq \sigma cs^{Allow}$$

Be Careful !!!!!

$$\sigma^{s}_{bot} = \frac{P_e}{A} + \frac{P_e(e_0)y_b}{I} - \frac{M_{DL}\,y_b}{I} - \frac{M_{SDL}\,y_b}{I} - \frac{M_{LL}\,y_b}{I} = \sigma_{ts}$$

$$\sigma^{s}_{bot} = \sigma_{ts} = \frac{P_e}{A} \times [1 + (\frac{e_0}{(-)k_t})] - \frac{M_{DL}}{S_b} - \frac{M_{SDL}}{S_b} - \frac{M_{LL}}{S_b} \geq \sigma^{Allow}_{ts}$$

Pre-Stressed Design – The Procedure

The pre-stressed concrete design start by suggesting cross sections that could be solution to the problem. Having these sections the next step will be to determine the required pre-stressing force and its eccentricity at the critical section. After the pre-stressing force is determined this pre-stressing force is used along the span and we must compute the eccentricity at other location so there is no allowable stress is exceeded. The common method is by computing the eccentricity envelope which is the maximum location that produces concrete stress less than allowable value. The spreadsheet that accompany this presentation will be used to clarify the whole concepts.

Pre-Stressed Concrete Design - Example

This example was taken from the Pre-stresses concrete Design Book by Antoine Naaman. In the example the author has used Z for the section modulus insteadof the typical mechanic of materials Books that normally use S.

Consider the pretensioned simply supported member shown in Fig. 4.15 with a span length of 70 feet. It is assumed that f'_c = 5000 psi, f'_{ci} = 4000 psi, $\bar{\sigma}_{ti}$ = -189 psi, $\bar{\sigma}_{ci}$ = 2400 psi, $\bar{\sigma}_{ts}$ = -424 psi, $\bar{\sigma}_{csus}$ = 2250 psi for sustained load, and $\bar{\sigma}_{cs}$ = 3000 psi for the maximum service load. Normal weight concrete is used, i.e., γ_c = 150 pcf, live load = 100 psf and superimposed dead load = 10 psf. Assume: $f_{pe} = 150$ ksi; $\eta = f_{pe} / f_{pi} = F / F_i = 0.83$; $f_{pi} = 180.723$ ksi; and $(e_o)_{mp} = y_b - 4 = 23.1$ in.

In order to calculate the stresses, the geometric properties of the section (given in Fig. 4.15) and the applied bending moments are needed.

- Minimum moment: $M_{min} = M_G = 0.573(70^2/8) = 350.962$ kips-ft
- Moment due to superimposed dead load: $M_{SD} = 0.04(70^2/8) = 24.5$ kips-ft
- Moment due to live load: $M_L = 0.4(70^2/8) = 245$ kips-ft
- Additional moment due to superimposed dead load and live load: $\Delta M = 0.44(70^2/8) = 269.5$ kips-ft
- Maximum moment: $M_{max} = M_{min} + \Delta M = 620.462$ kips-ft
- Sustained moment: $M_{sus} = M_G + M_{SD} = 375.462$ kips-ft

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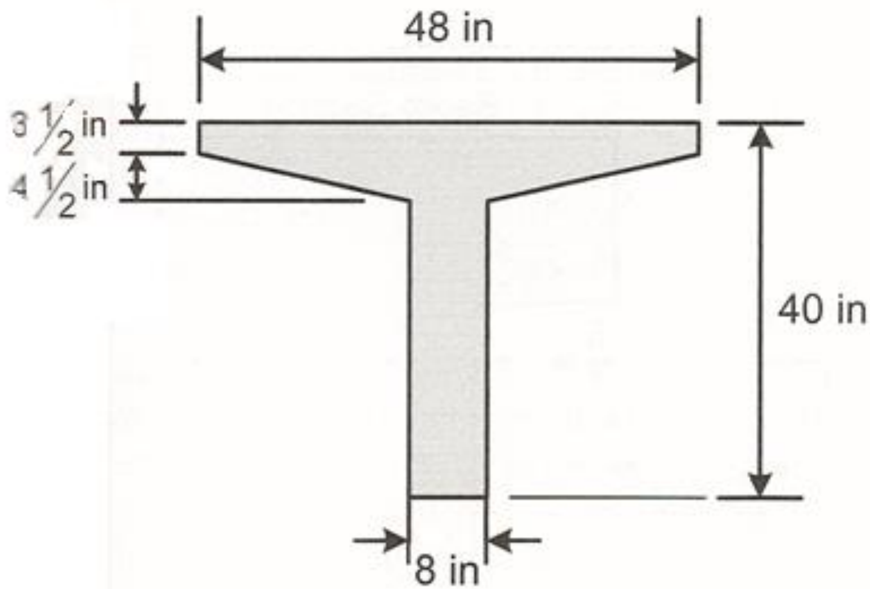
Pre-Stressed Concrete Design - Example



SECTION PROPERTIES

$A = 550\ in^2$

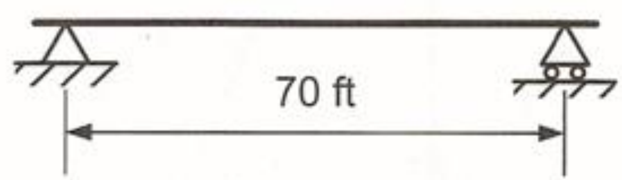
$I = 82{,}065\ in^4$

$y_t = 12.9\ in;\quad y_b = 27.1\ in$

$Z_t = 6362\ in^3;\quad Z_b = 3028\ in^3$

$k_t = -5.51\ in;\quad k_b = 11.57\ in$

$w_G = 0.573\ klf$



(a) Investigate flexural stresses at midspan given: $F = 229.5$ kips (corresponding to ten ½-in diameter strands), $F_i = F/\eta = 276.5$ kips, and $e_o = 23.1$ in.

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Pre-Stressed Concrete Design - Example

(a) Investigate flexural stresses at midspan given: $F = 229.5$ kips (corresponding to ten ½-in diameter strands), $F_i = F/\eta = 276.5$ kips, and $e_o = 23.1$ in.

Referring to the four stress inequality equations given in Table 4.2 (way 1) and multiplying the values of moments by 12,000 in order to have units of pounds per square inch leads to:

Condition I: $$\sigma_{ti} = \frac{F_i}{A_c}\left(1 - \frac{e_o}{k_b}\right) + \frac{M_{min}}{Z_t} \geq \bar{\sigma}_{ti}$$

$$\sigma_{ti} = \frac{276500}{550}\left(1 - \frac{23.1}{11.57}\right) + \frac{350.962 \times 12000}{6362} \simeq 161 \text{ psi} > \bar{\sigma}_{ti} = -189 \text{ psi} \qquad \text{OK}$$

The results for the other conditions are given as follows:

Condition II: $\sigma_{ci} = 1219 \text{ psi} < \bar{\sigma}_{ci} = 2400 \text{ psi}$ OK

Condition III: $$\begin{cases} \sigma_{cs} = 754 \text{ psi} < \bar{\sigma}_{cs} = 3000 \text{ psi for } M_{max} \\ \text{or} \\ \sigma_{csus} = 292 \text{ psi} < \bar{\sigma}_{csus} = 2250 \text{ psi for } M_{sus} \end{cases} \qquad \text{OK}$$

Condition IV: $\sigma_{ts} = -292 \text{ psi} > \bar{\sigma}_{ts} = -424 \text{ psi}$ OK

Therefore the section is satisfactory with respect to flexural stresses.

(b) Plot the feasibility domain for the above problem and check geometrically if allowable stresses are satisfied.

The equations at equality given in Table 4.2 (way 2) are used to plot linear relationships of e_o versus $1/F_i$ on Fig. 4.16. They are reduced to the following convenient form, the first of which is detailed:

Condition I: $$e_o \leq k_b + (1/F_i)(M_{\min} - \bar{\sigma}_{ti} Z_t) = 11.57 + (1/F_i)(350.962 \times 12000 + 189 \times 6362)$$

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Pre-Stressed Concrete Design - Example

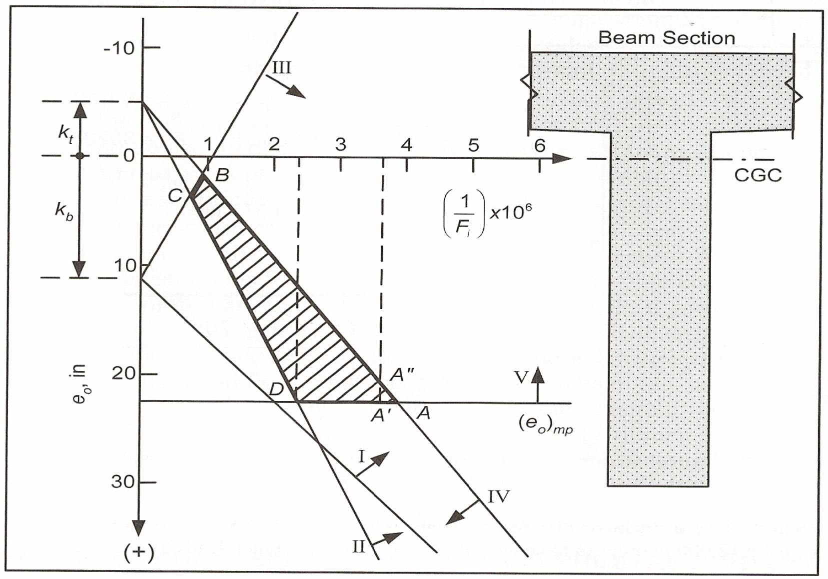

Figure 4.16 Feasibility domain for example 4.9.1.

which can be put in the following convenient form, for this as well as for the other conditions:

Condition I: $$e_o = 11.57 + 5.410\left(\frac{10^6}{F_i}\right)$$

Condition II: $$e_o = -5.51 + 11.4787\left(\frac{10^6}{F_i}\right)$$

Condition III: $$\begin{cases} e_o = 11.57 - 14.024\left(\frac{10^6}{F_i}\right) & \text{for } M_{max} \\ e_o = 11.57 - 11.818\left(\frac{10^6}{F_i}\right) & \text{for } M_{sus} \Rightarrow \text{Controls} \end{cases}$$

Condition IV: $$e_o = -5.51 + 7.424\left(\frac{10^6}{F_i}\right)$$

Pre-Stressed Concrete Design - Example

where e_o is in inches and F_i is in pounds. Also equation V showing $(e_o)_{mp} = 23.1$ in is plotted in Fig. 4.16. The five lines delineate a feasibility domain *ABCD*.

Let us check if the given values of F_i and e_o are represented by a point which belongs to the feasible region:

$$\frac{1}{F_i} = \frac{1}{276{,}500} = 3.6 \times 10^{-6}$$

The representative point is shown in Fig. 4.16 as point *A'*. Since it is on line *AD*, it belongs to the feasible region and therefore all allowable stresses are satisfied. Note that all stresses would still be satisfied if the eccentricity is reduced to approximately 21 in for the same force. This is shown as point *A"* on line *AB* and allows the designer to accept a reasonable tolerance on the value of e_o actually achieved during the construction phase.

(c) Assuming the prestressing force is not given, determine its design value and corresponding eccentricity.

This is essentially a typical design problem where the concrete cross section is given. It can be solved directly analytically or from the graphical representation of the feasibility domain. In any case, the graphical representation helps in the analytical solution. It dictates the choice of point *A* of Fig. 4.16 as the solution that minimizes the prestressing force. Point *A* corresponds to the intersection of line V representing $(e_o)_{mp}$ with that representing stress condition IV. The corresponding value of *F* is obtained by replacing e_o by $(e_o)_{mp}$ in Eq. IV (way 3) of Table 4.2; that is:

$$F = \frac{M_{\max} + \bar{\sigma}_{ts} Z_b}{(e_o)_{mp} - k_t} = \frac{620.462 \times 12000 - 424 \times 3028}{23.1 + 5.51} = 215{,}368 \text{ lb}$$

and

$$F_i = \frac{F}{0.83} = 259{,}479 \text{ lb} \simeq 259.5 \text{ kips}$$

Pre-Stressed Concrete Design - Example

Graphically the coordinates of A can be read in Fig. 4.16 as $e_o = 23.1$ in and $1/F_i = 3.9 \times 10^{-6}$ which leads to $F_i \approx 257{,}000$ lb = 257 kips. It can be seen that the graphical solution gives essentially the same answer as the analytical one. Note that the practical value of the prestressing force to use in the design should correspond to an integer number of tendons. In this case, exactly 9.38 strands each with a final force of 22.95 kips would be required. The number is rounded off to 10. The resulting higher prestressing force allows for an acceptable tolerance on the value of e_o, which can be varied now from 23.1 in to 21.33 in (see Table 4.5).

(d) If the beam is to be used with different values of live loads, what is the maximum value of live load it can sustain?

Referring to the stress inequality conditions, it can be observed that conditions I and II (which do not depend on the live load moment) do not change and therefore lines I and II of Fig. 4.16 are fixed. Increasing the value of the live load will increase the value of M_{max} and thus will change the slopes of lines III and IV so as to reduce the size of the feasible domain. Consequently, point A of the feasible domain will move in the direction of AD and line BA tends to rotate (about the intercept point k_t) toward CD. Similarly line III will rotate about the intercept, k_b, towards line I. The maximum value of live load correspond to the line that merges first with the other one. In this case, it is the live load that will make lines II and IV coincide or have same slopes. Therefore:

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Pre-Stressed Concrete Design - Example

$$\frac{M_{max} + \bar{\sigma}_{ts} Z_b}{\eta} = 11.4787 \times 10^6$$

which leads to M_{max} = 10,811,193 lb-in = 900.93 kips-ft. Subtracting from $M_{\max}$ the values of moments due to dead load and superimposed dead load (375.462 kips-ft), leads to a live load moment of 525.468 kips-ft, from which the live load can be determined as 858 plf or 214.5 psf. The representative point in Fig. 4.16 is D which shows the following coordinates: $e_o = 23.1$ in and $10^6/F_i \simeq 2.5$, i.e., $F_i \simeq$ 400,000 lb = 400 kips. The reader is encouraged to check numerically in this case that the two allowable stresses $\bar{\sigma}_{ti}$ and $\bar{\sigma}_{ts}$ are attained exactly while the two others are satisfied, as indicated by the geometric representation. Note that such a design may have to be revised if the assumed value of e_o cannot be practically achieved. Note also that while the limit capacity of this prestressed beam in now attained from an allowable stress point of view, it can still be designed to carry a larger live load should partial prestressing be considered.

ABC: Vigas Pretensadas para Puentes Principios Básicos de Diseño

Ultimate Flexural Design Capacity

Load Combinations

- $U = 1.4\,(D + F)$
- $U = 1.2\,(D + F + T) + 1.6\,(L + H) + 0.5\,(L_r \text{ or } S \text{ or } R)$
- $U = 1.2D + 1.6\,(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$
- $U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
- $U = 1.2D + 1.0E + 1.0L + 0.2S$
- $U= 0.9D + 1.6W + 1.6H$
- $U= 0.9D + 1.0E + 1.6H$

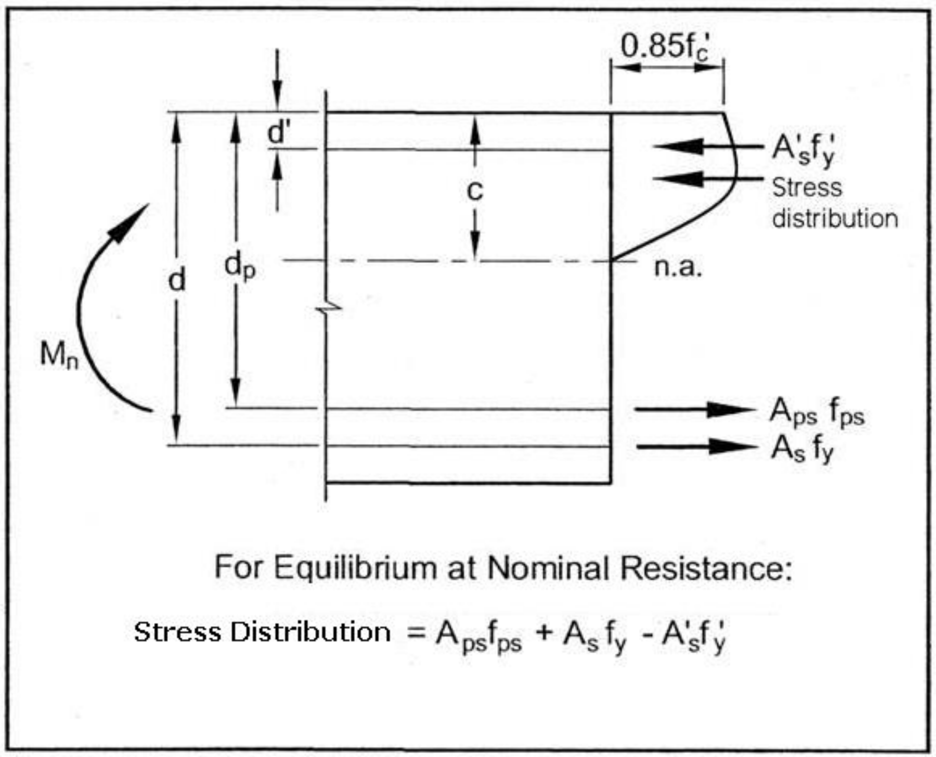
Strength Design

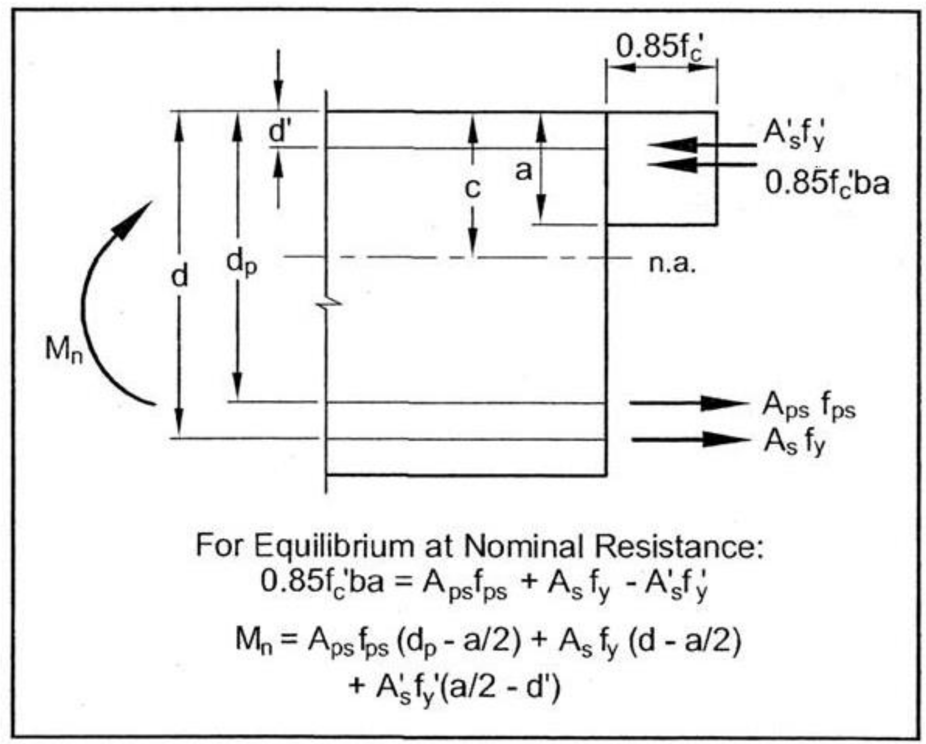
- Strength design is based using the rectangular stress block
- The stress in the pre-stressing steel at nominal strength, f_{ps}, can be determined by strain compatibility or by an approximate empirical equation
- For elements with compression reinforcement, the nominal strength can be calculated by assuming that the compression reinforcement yields. Then verified.
- The designer will normally choose a section and reinforcement and then determine if it meets the basic design strength requirement:

$$\phi M_n \geq M_u$$

Stress Block Theory

- The Whitney stress block is a simplified stress distribution that shares the same centroid and total force as the real stress distribution





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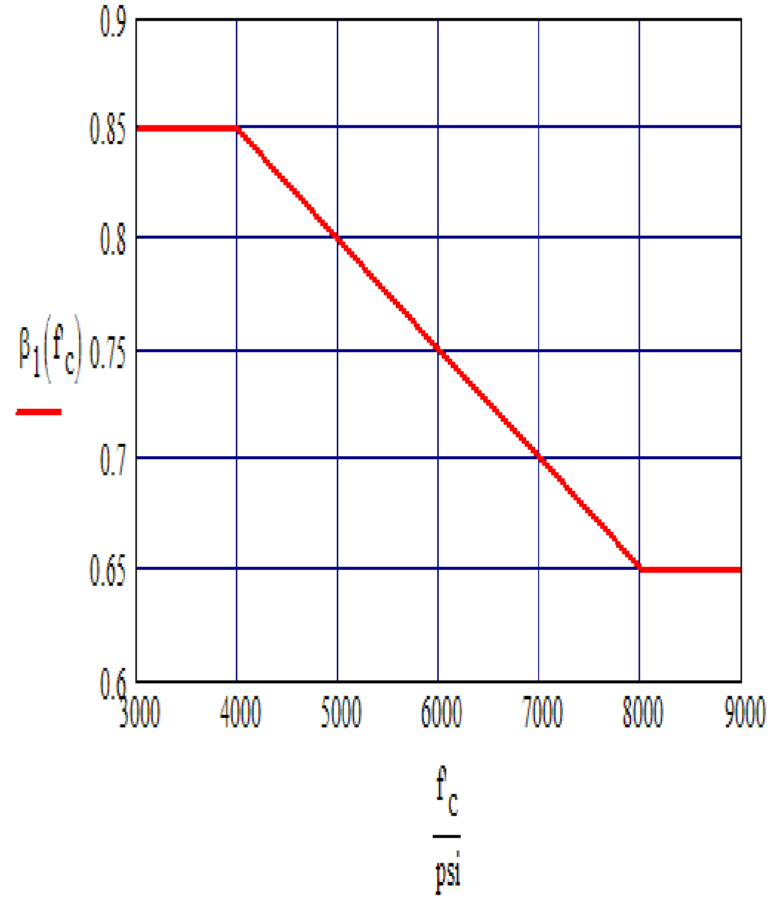
Equivalent Stress Block – b_1 Definition

$\beta_1 = 0.85$

when $f'_c <= 4{,}000$ psi

$\beta_1 = .85 – 0.05 \{f'c\ (ksi) – 4(ksi)\}$

$\beta_1 = .65$ when $f'_c > 8{,}000$ psi



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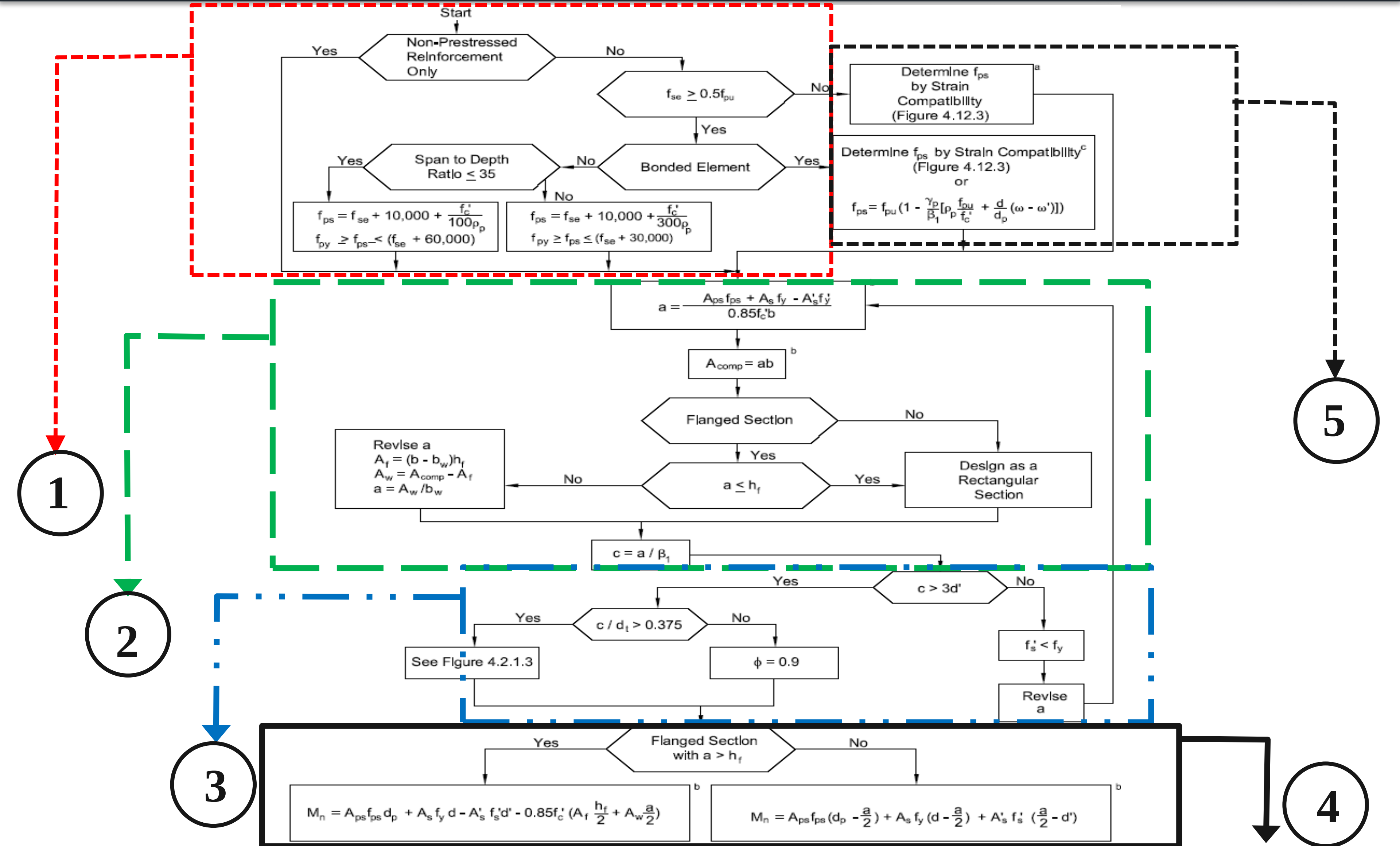
Strength Design Flowchart



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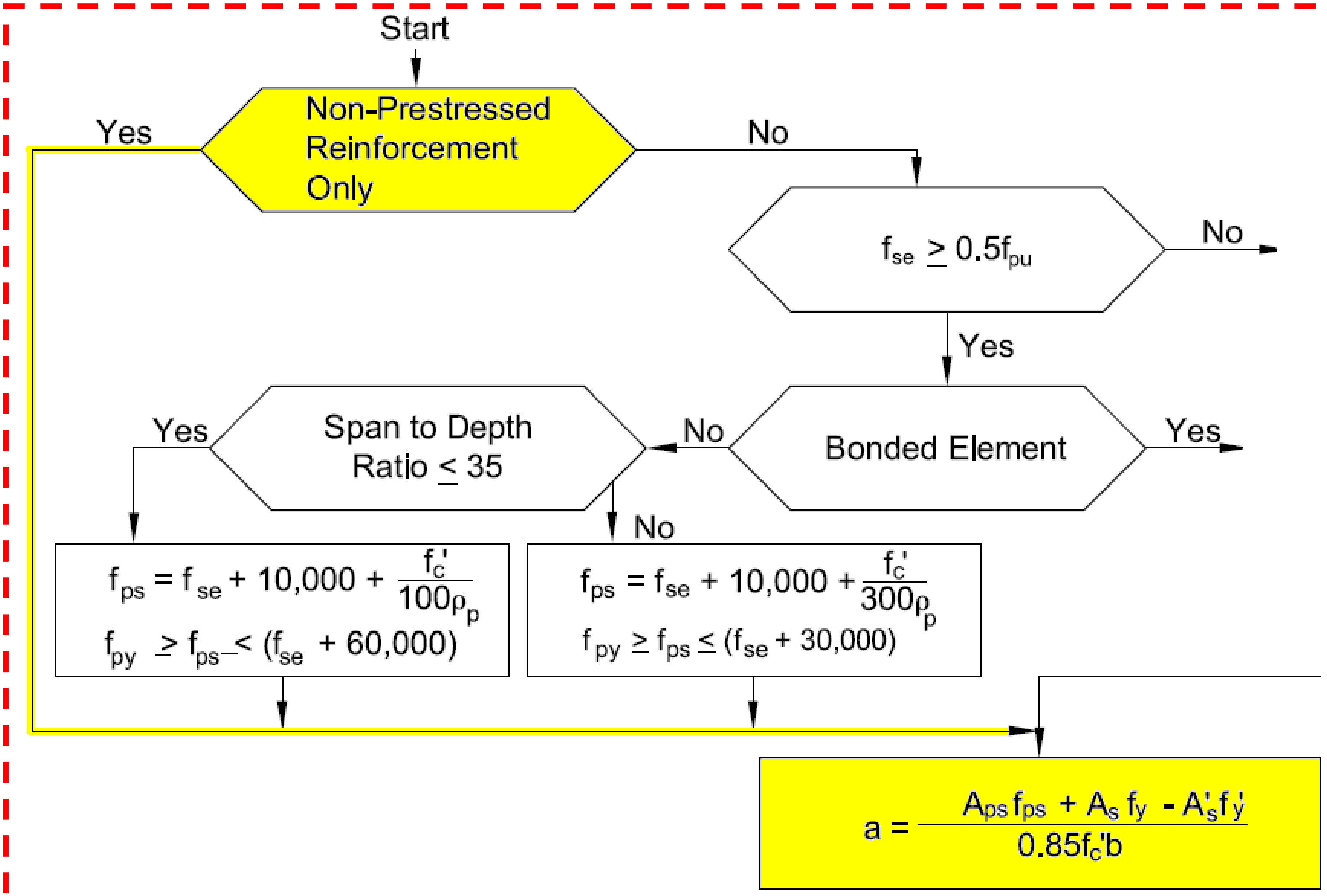
Find depth of compression block

1



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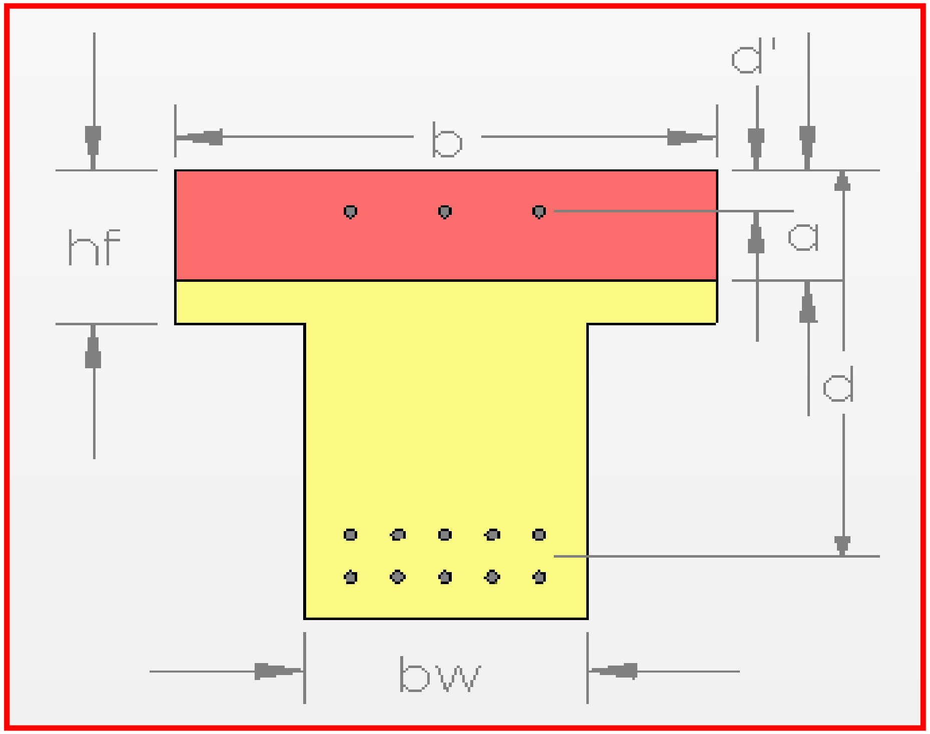
Depth of Compression Block



$$a = \frac{A_s \cdot f_y - A'_s \cdot f'_y}{.85 \cdot f'_c \cdot b}$$

Assumes compression steel yields

Where:

- A_s is the area of tension steel
- A'_s is the area of compression steel
- f_y is the mild steel yield strength

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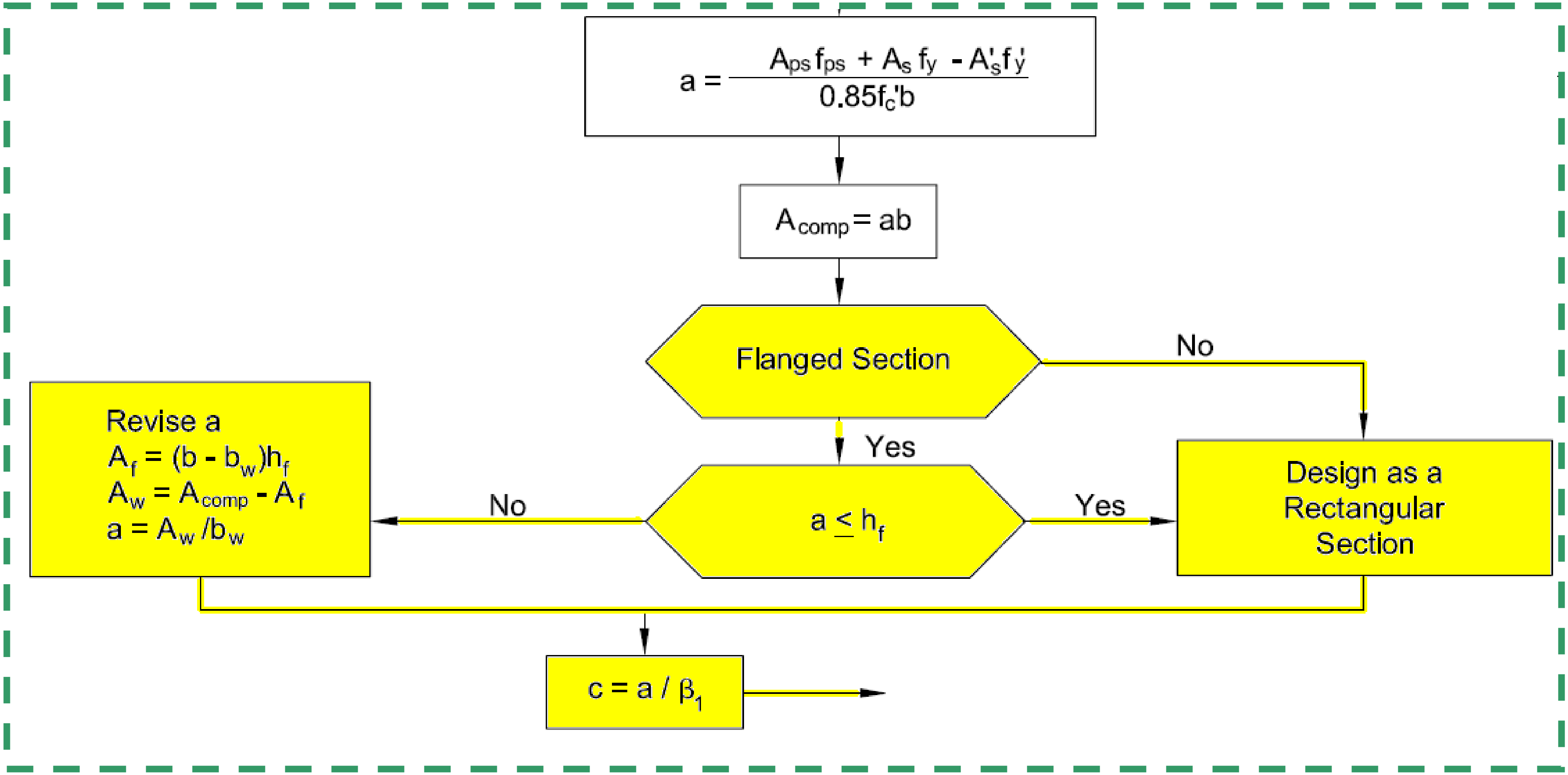
Flanged Sections

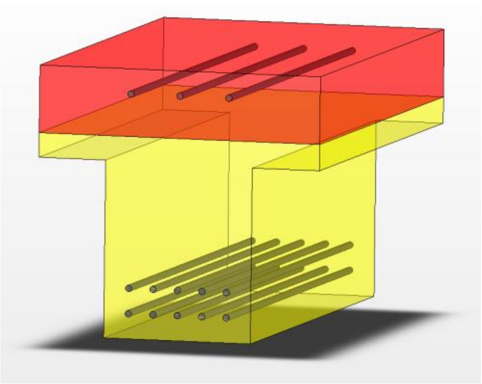
2

Checked to verify that the compression block is truly rectangular



Compression Block Area

- If compression block is rectangular, the flanged section can be designed as a rectangular beam



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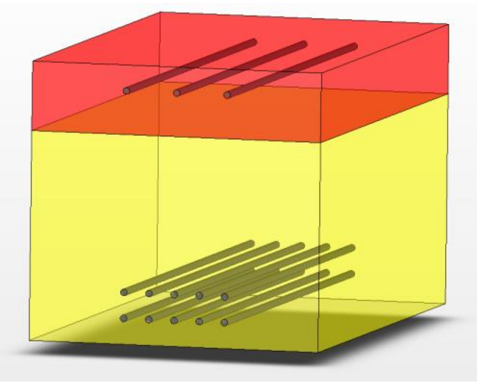

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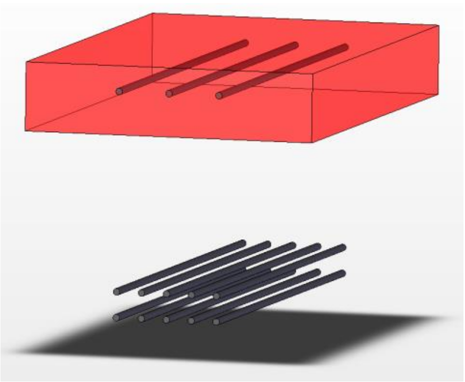

$$A_{comp} = a \cdot b$$

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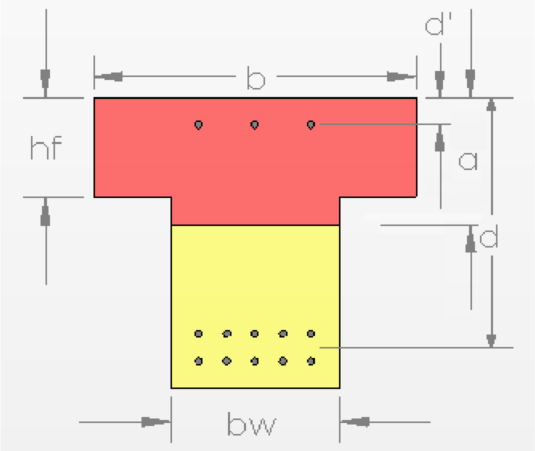
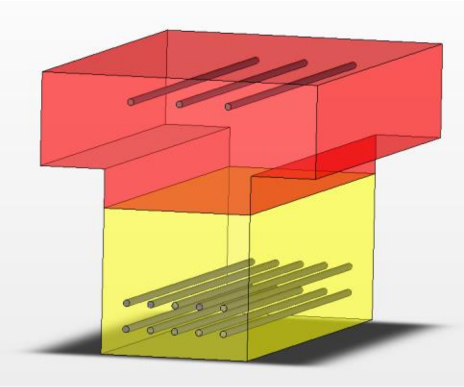
Compression Block Area

- If the compression block is not rectangular ($a > h_f$),





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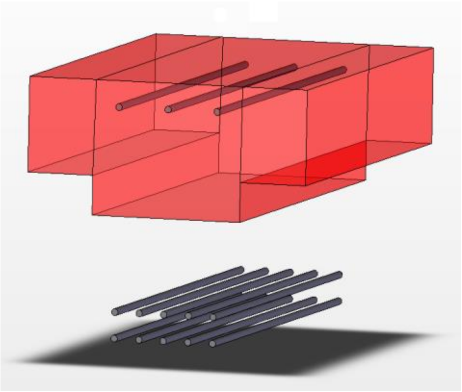

to find "a"

$$A_f = (b - b_w) \cdot h_f$$

$$A_w = A_{comp} - A_f$$

$$a = \frac{A_w}{b_w}$$

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Determine Neutral Axis

- From statics and strain compatibility

$$\boxed{c = a / \beta_1}$$

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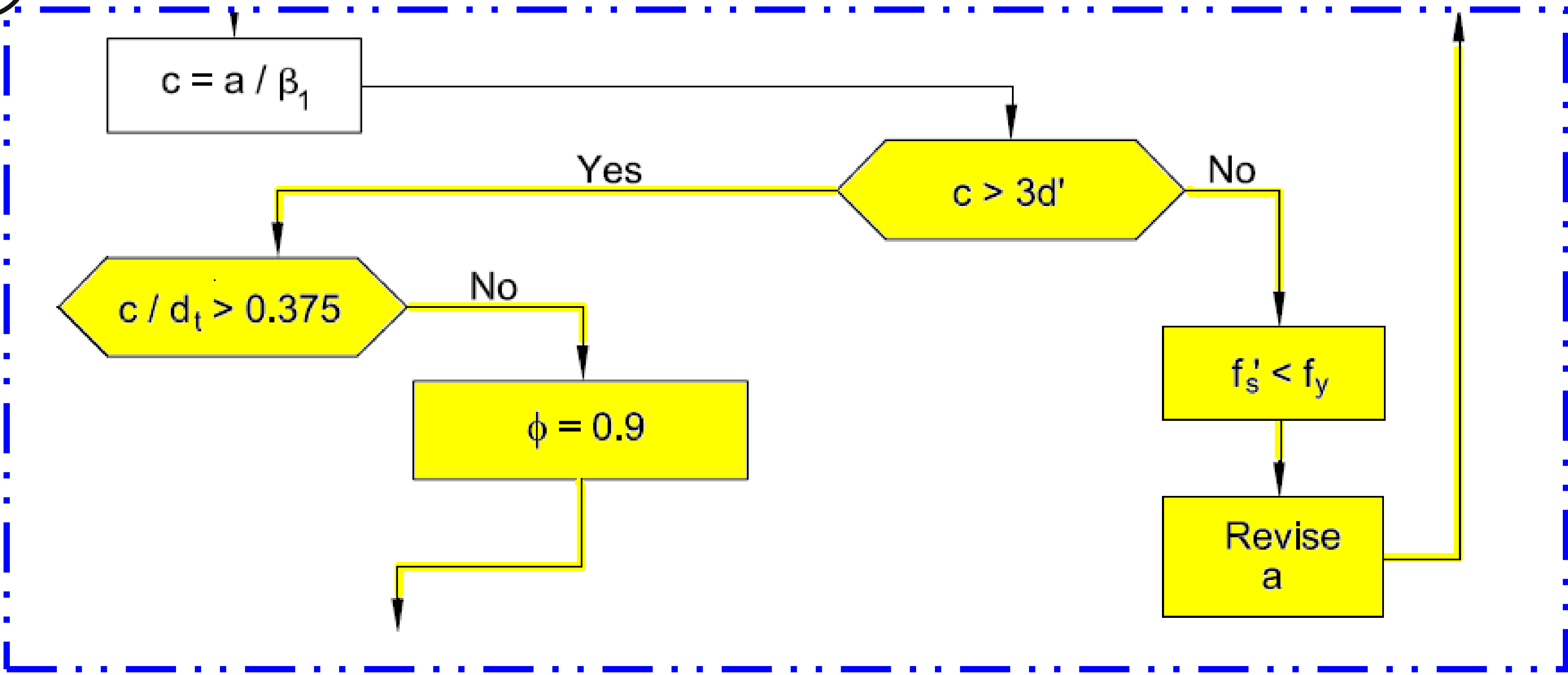
Check Compression Steel

3



- Verify that compression steel has reached yield using strain compatibility

Compression Comments

- By strain compatibility, compression steel yields if:

$$c > 3 \cdot d'$$

- If compression steel has not yielded, calculation for "a" must be revised by substituting actual stress for yield stress
- Non prestressed members should always be tension controlled, therefore $c / d_t < 0.375$
- Add compression reinforcement to create tesnion controlled secions

Moment Capacity

- 2 equations
  - rectangular stress block in the flange section
  - rectangular stress block in flange and stem section

4

Flanged Section with $a > h_f$

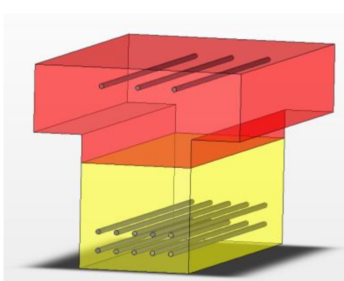
Yes:

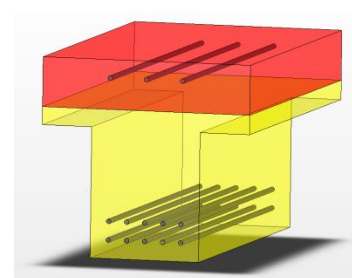
$$M_n = A_{ps} f_{ps} d_p + A_s f_y d - A'_s f'_s d' - 0.85 f'_c \left(A_f \frac{h_f}{2} + A_w \frac{a}{2}\right)$$

b

No:

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2}\right) + A_s f_y \left(d - \frac{a}{2}\right) + A'_s f'_s \left(\frac{a}{2} - d'\right)$$





PRE-STRESSED CONCRETE SEMINAR

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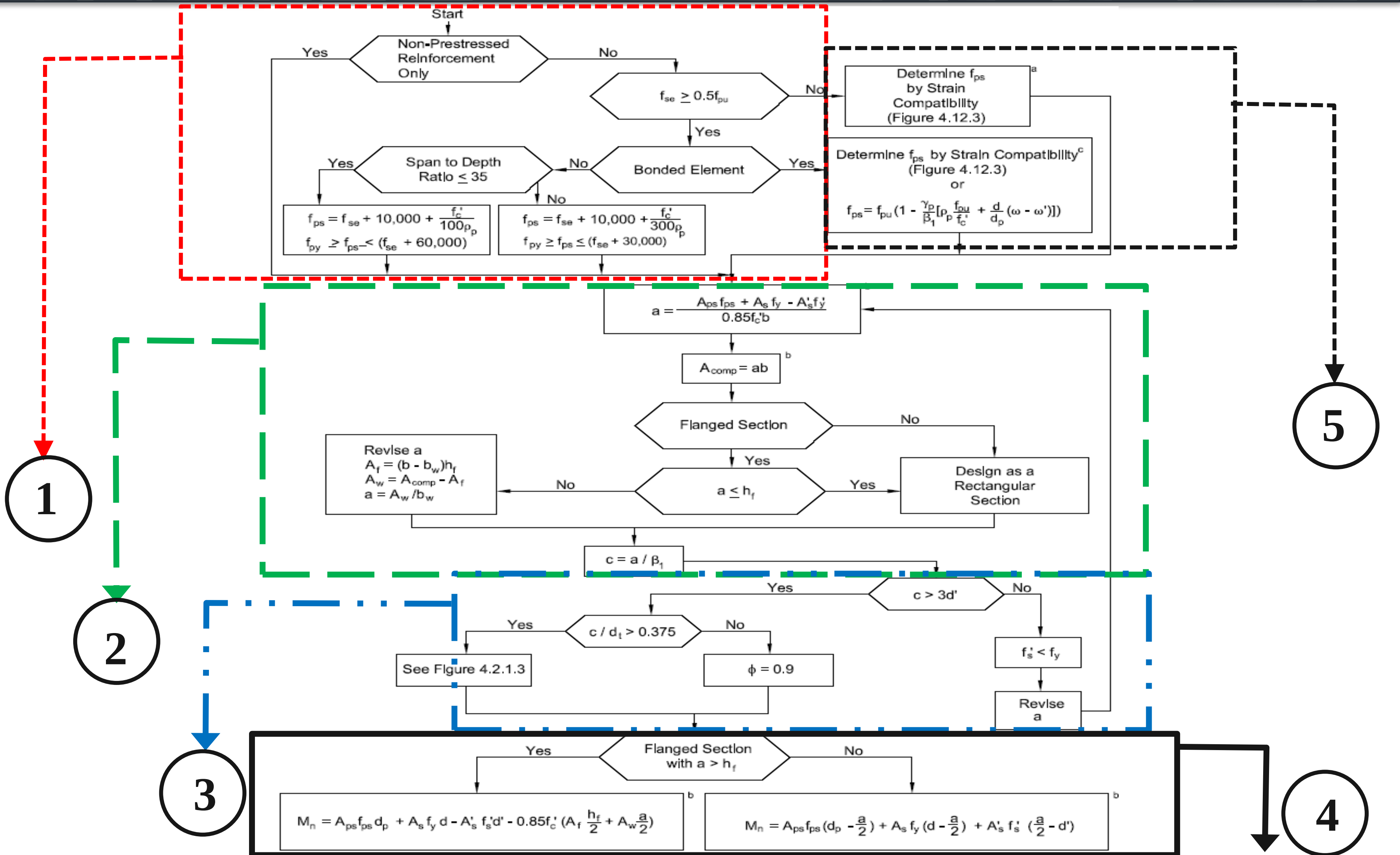
Strength Design Flowchart



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Stress in Strand

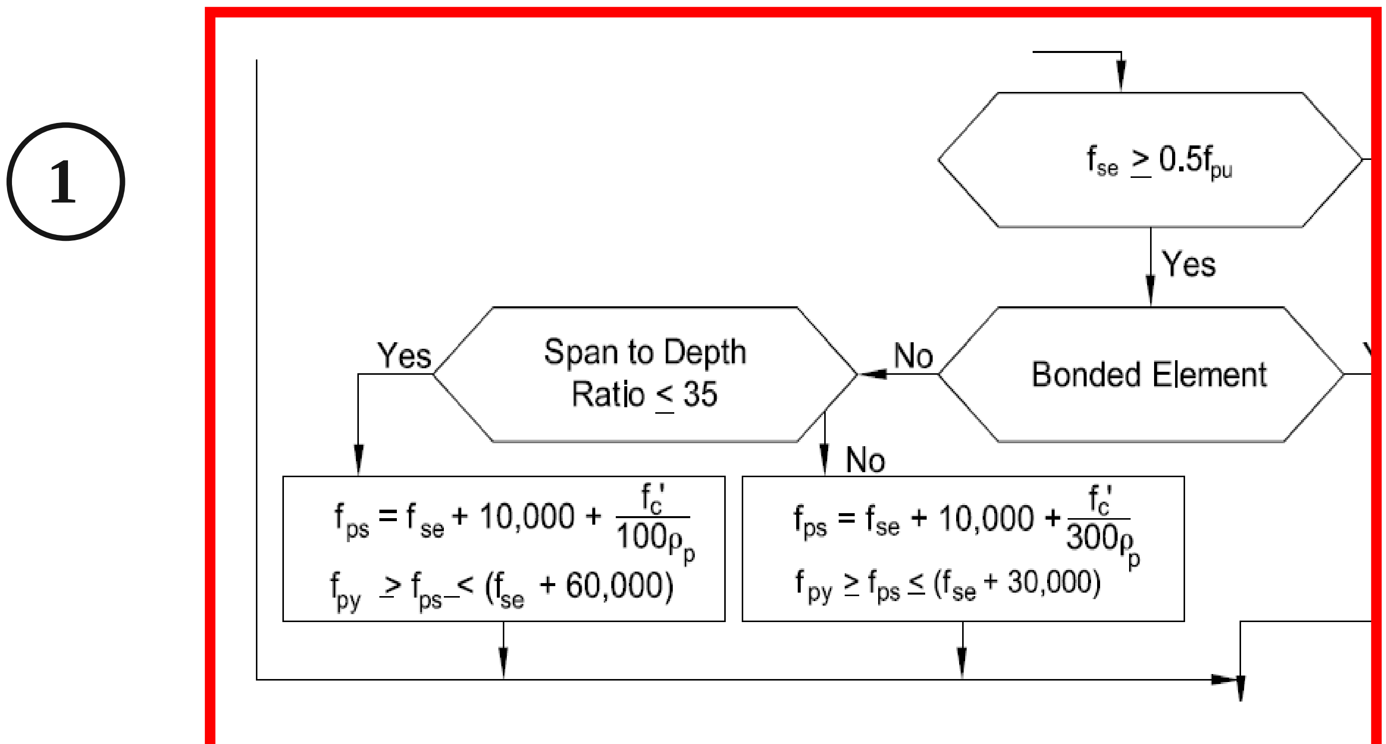
f_{se} - stress in the strand after losses

f_{pu} - is the ultimate strength of the strand

f_{ps} - stress in the strand at nominal strength

$\rho_p = (A_{ps})/(bd_p)$ Pre-stressed Reinforcement

This portion of the flowchart is dedicated to determining the stress in the prestress reinforcement



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Stress in Strand

- Typically the jacking force is 65% or greater
- The short term losses at midspan are about 10% or less
- The long term losses at midspan are about 20% or less

$$f_{se} \geq 0.5 \cdot f_{pu}$$

The most commonly used prestressing material in the United States is Grade 270 ksi low-relaxation, seven-wire strand, defined by ASTM A 416. The most common size is 0.5-in., although there is increasing use of 0.6-in. strand, especially for post-tensioning of large scale projects. The properties of these strands are as follows:

Nominal Diameter, in.	0.5	0.6
Area, sq. in.	0.153	0.217
Tensile strength f_{pu}, ksi	270	270
Tensile force capacity, kips	41.3	58.6
Jacking stress, ksi = $0.75\ f_{pu}$	202.5	202.5

PRE-STRESSED CONCRETE SEMINAR

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Stress in Strand

f_{pu} - is the ultimate strength of the strand

f_{ps} - stress in the strand at nominal strength

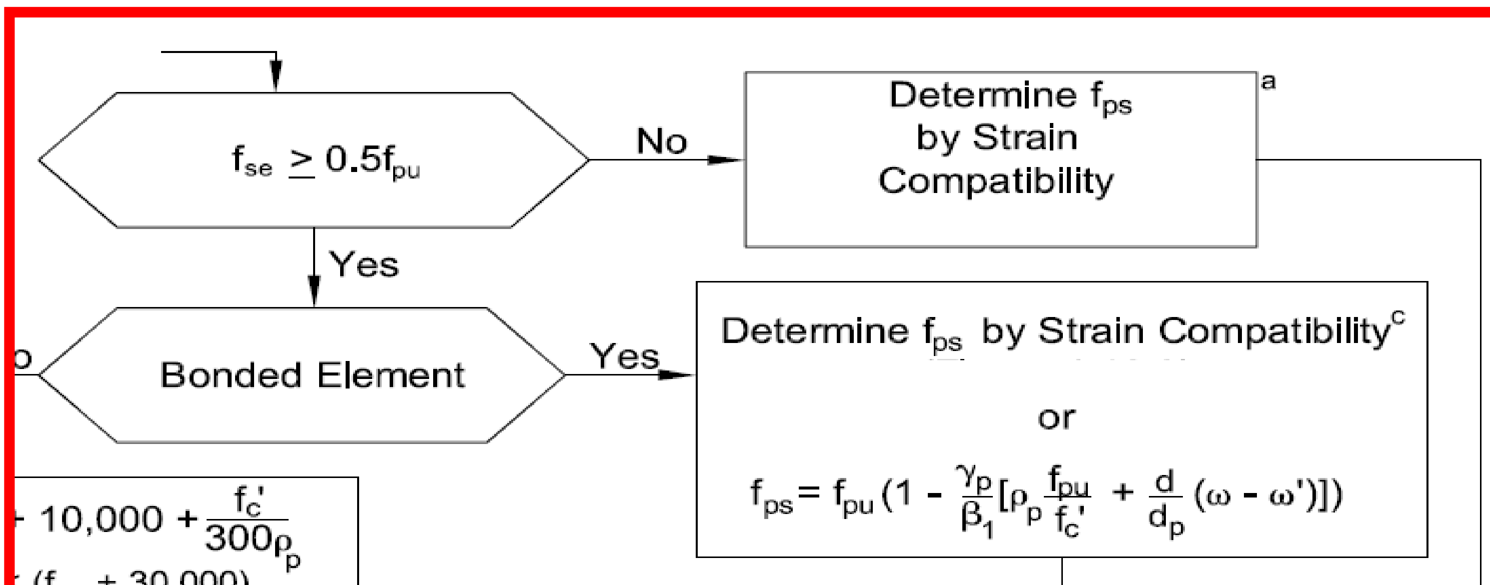
$\omega = (A_s{*}f_y)/(\,bdf'_c)$ Non Pre-stressed Reinforcement

$\omega' = (A'_s{*}f'_y)/(\,bdf'_c)$ Non Pre-stressed Reinforcement

$\rho_p = (A_{ps})/(\,bd_p)$ Pre-stressed Reinforcement

This portion of the flowchart is dedicated to determining the stress in the prestress reinforcement

5



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Stress in Strand

- Prestressed Bonded reinforcement

$$f_{ps} = f_{pu} \cdot \left(1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \cdot \frac{f_{pu}}{f'_c} + \frac{d}{d_p}\left(\omega - \omega'\right)\right]\right)$$

γ_p = factor for type of prestressing strand, see ACI 18.0

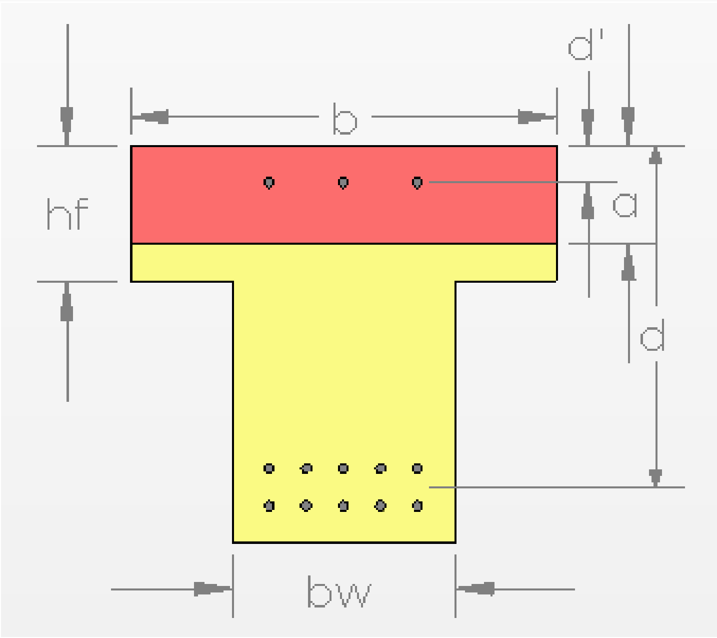
= .55 for f_{py}/f_{pu} not less than .80

= .45 for f_{py}/f_{pu} not less than .85

= .28 for f_{py}/f_{pu} not less than .90 (Low Relaxation Strand)

ρ_p = prestressing reinforcement ratio

Compression Block Height



Assumes compression steel yields

$$a = \frac{A_{ps} \cdot f_{ps} + A_s \cdot f_y - A'_s \cdot f'_y}{.85 \cdot f'_c \cdot b}$$

Prestress component

Where

A_{ps} - area of prestressing steel

f_{ps} - prestressing steel strength

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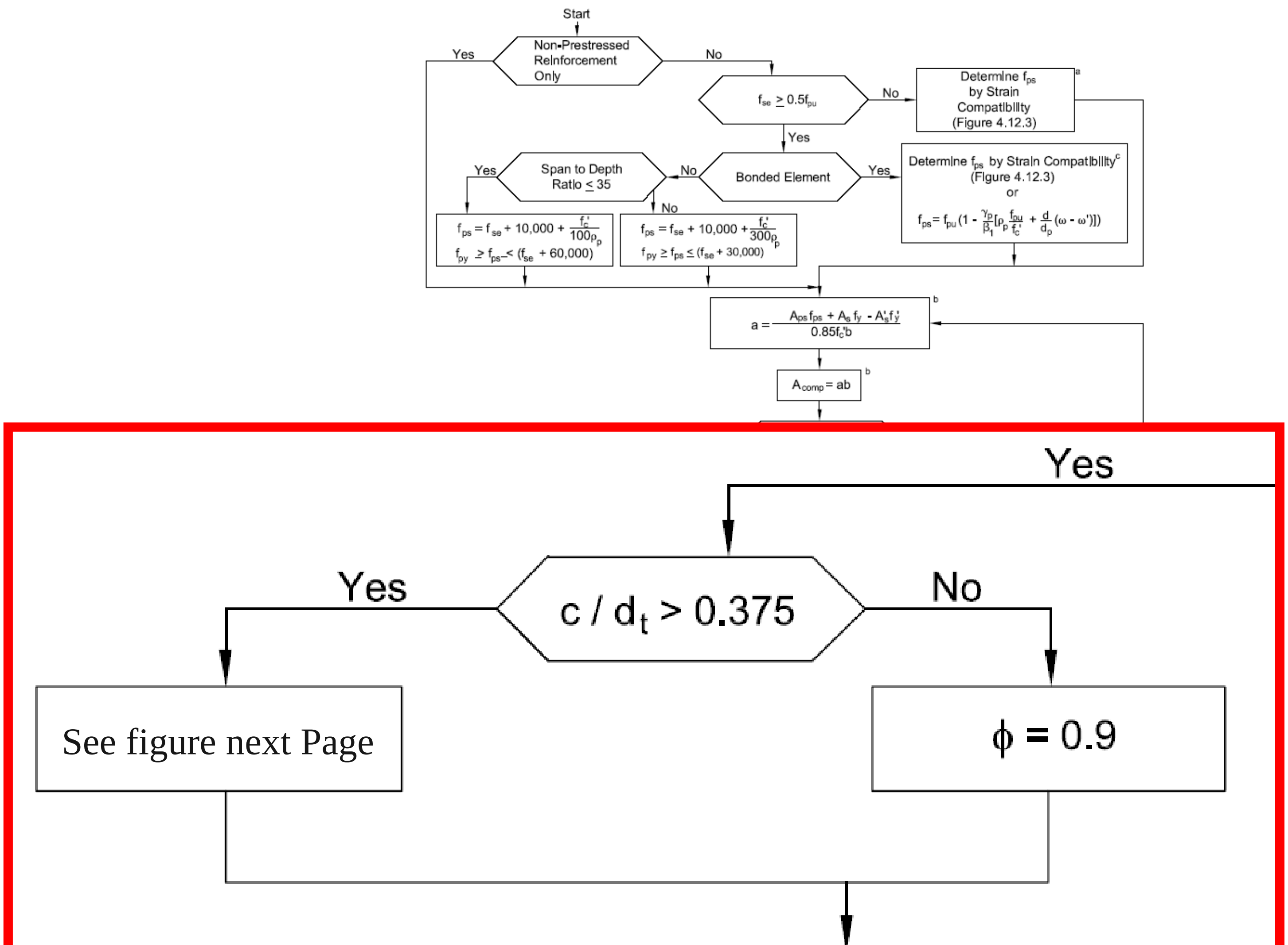
Flexural Strength Reduction Factor

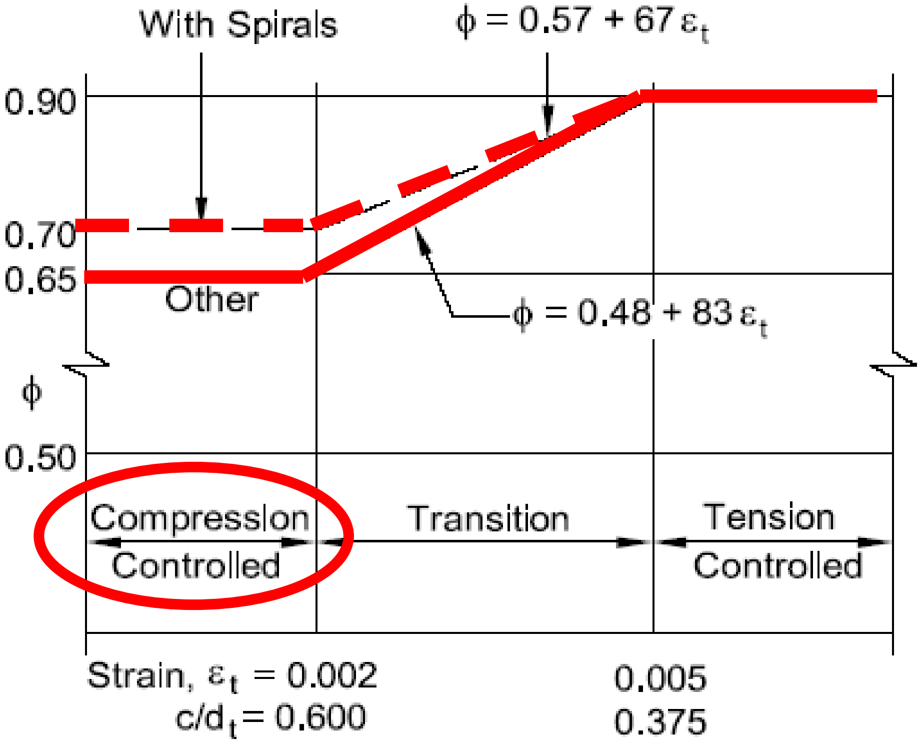


Flexural Strength Reduction Factor

- Based on primary reinforcement strain
- Strain is an indication of failure mechanism
- Three Regions

c/d_t = > 0.6
ϕ = 0.70 with spiral ties
ϕ = 0.65 with stirrups

c/d_t <= 0.375
ϕ = 0.90 with spiral ties or stirrups



Transition Zone

- $0.002 < \varepsilon < 0.005$ at extreme steel tension fiber, or
- $0.375 < c/d_t < 0.6$

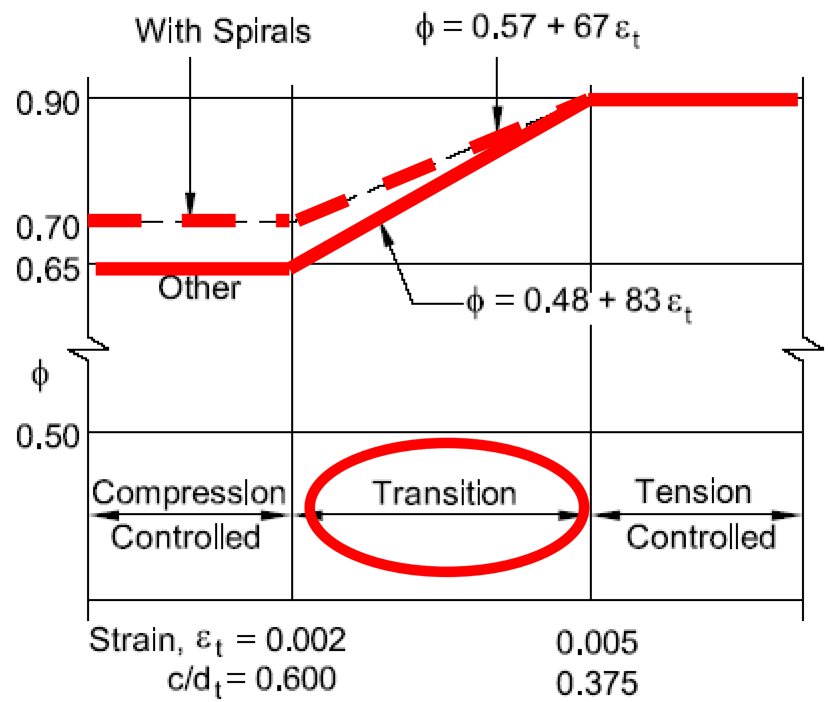
$\phi = 0.57 + 67(\varepsilon)$ or

$\phi = 0.48 + 83(\varepsilon)$ with spiral ties

$\phi = 0.37 + 0.20/(c/d_t)$ or

$\phi = 0.23 + 0.25/(c/d_t)$

ϕ with stirrups



Strand Slip Regions

- ACI Section 9.3.2.7

  ‘*where the strand embedment length is less than the development length*’

  $\phi = 0.75$

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Limits of Reinforcement

- To prevent failure immediately upon cracking, Minimum A_s is determined by:

$$A_{s,min} = \frac{3 \cdot \sqrt{f'_c}}{f_y} \cdot b_w \cdot d \geq \frac{200 \cdot b_w \cdot d}{f_y}$$

- $A_{s,min}$ is allowed to be waived if tensile reinforcement is 1/3 greater than required by analysis

Limits of Reinforcement

- The flexural member must also have adequate reinforcement to resist the cracking moment
  - Where

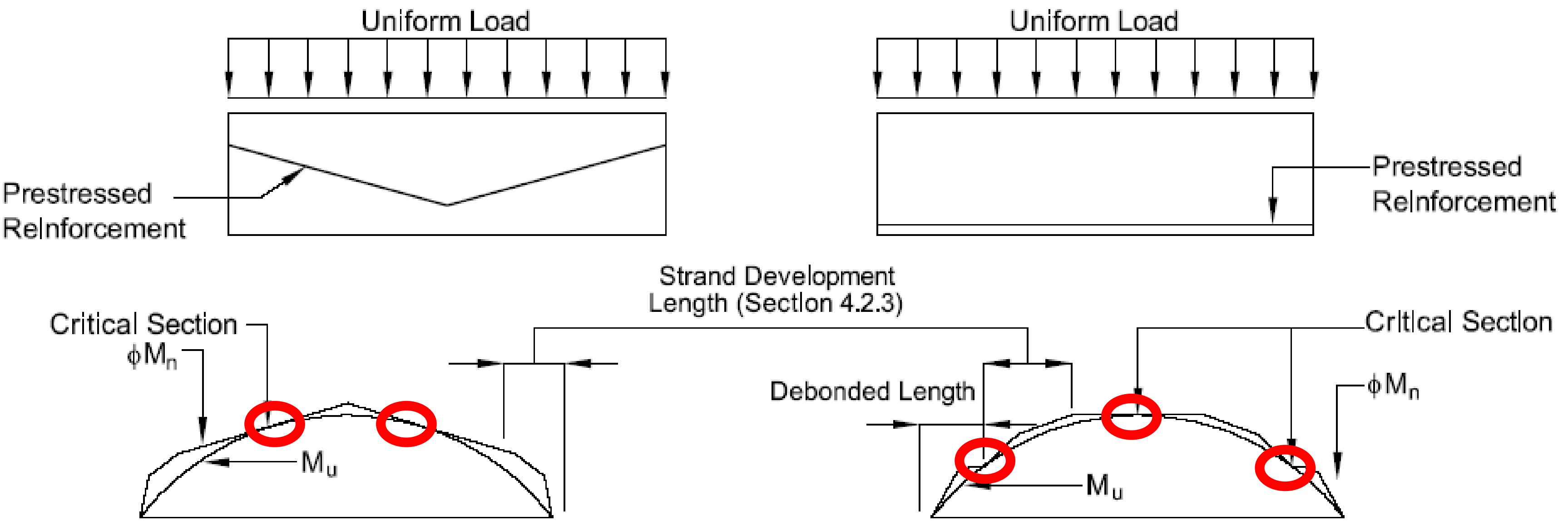
$$\phi M_n \geq 1.2 M_{cr}$$

$$M_{cr} = S_{bc} \cdot \left[\frac{P}{A} + \frac{Pe}{S_b} + f_r \right] - M_{nc} \left[\frac{S_{bc}}{S_b} - 1 \right]$$

Section after composite has been applied, including prestress forces

Correction for initial stresses on non-composite, prior to topping placement

Critical Sections



(a) Beam with Strands Depressed at Midpoint – Uniform Load

(b) Beam with Straight Strands - Some Debonded Near End

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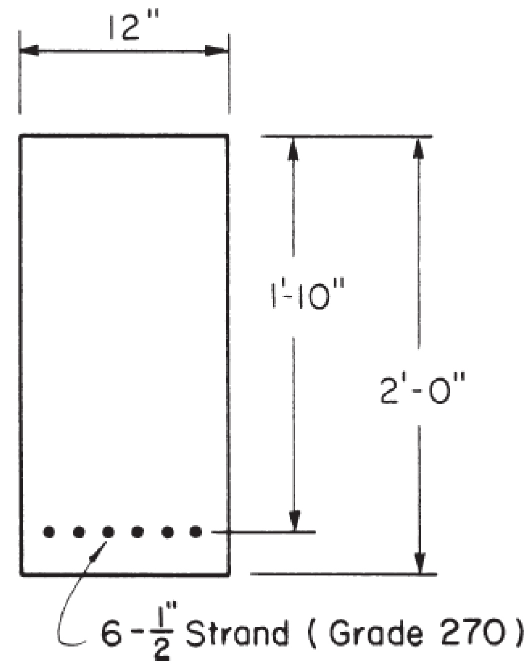
Example from PCA

Example 24.3—Flexural Strength of Prestressed Member Using Approximate Value for f_{ps}

Calculate the nominal moment strength of the prestressed member shown.

$f'_c = 5000$ psi

$f_{pu} = 270{,}000$ psi (low-relaxation strands; $f_{py} = 0.90 f_{pu}$)



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Example from PCA

Calculations and Discussion	Code Reference
1. Calculate stress in prestressed reinforcement at nominal strength using approximate value for f_{ps}. For a fully prestressed member, Eq. (18-3) reduces to:	
$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right)$$	Eq. (18-3)
$$= 270\left(1 - \frac{0.28}{0.80} \times 0.00348 \times \frac{270}{5}\right) = 252 \text{ ksi}$$	
where	
$$\gamma_p = 0.28 \text{ for } \frac{f_{py}}{f_{pu}} = 0.90 \text{ for low-relaxation strand}$$	
$$\beta_1 = 0.80 \text{ for } f'_c = 5000 \text{ psi}$$	10.2.7.3
$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{6 \times 0.153}{12 \times 22} = 0.00348$$	

Example from PCA

Example 24.3 (cont'd)	Calculations and Discussion	Code Reference

2. Calculate nominal moment strength from Eqs. (9) and (10) of Part 24

Compute the depth of the compression block:

$$a = \frac{A_{ps}f_{ps}}{0.85bf'_c} = \frac{0.918 \times 252}{0.85 \times 12 \times 5} = 4.54 \text{ in.}$$ Eq. (10)

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)$$ Eq. (9)

$$M_n = 0.918 \times 252\left(22 - \frac{4.54}{2}\right) = 4565 \text{ in-kips} = 380 \text{ ft-kips}$$

3. Check if tension controlled — 10.3.4

$$c/d_p = (a/\beta_1)/d_p = \left(\frac{4.54}{0.80}\right)/22$$

$$c/d_p = 0.258 < 0.375$$ R9.3.2.2

Tension controlled $\phi = 0.9$