Train Position Measurement in Tren Urbano Project

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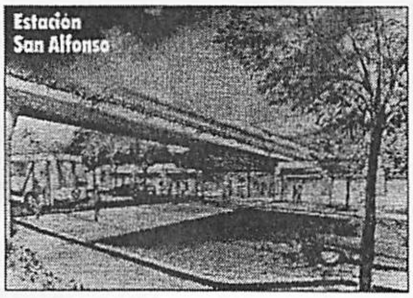
Problem

- Requirement of Headways is:
  - 4 minutes in rush hours
  - 5 minutes in basic service
  - 12 minutes in evenings, weekends, and holiday

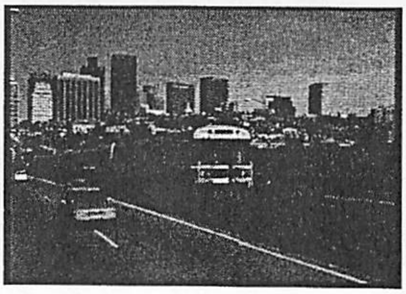
Problem

- Measuring
  - Position
  - Speed
- Keeping in mind that trains could slip while
  - Accelerating
  - Braking



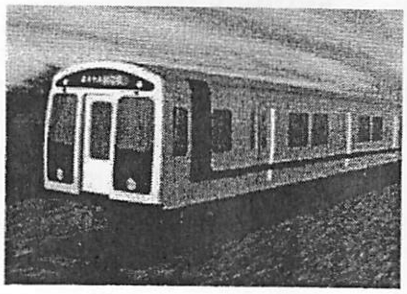
Objectives

- Obtain an optimal estimate of the train position and speed given measurements of wheel rotation and train acceleration.

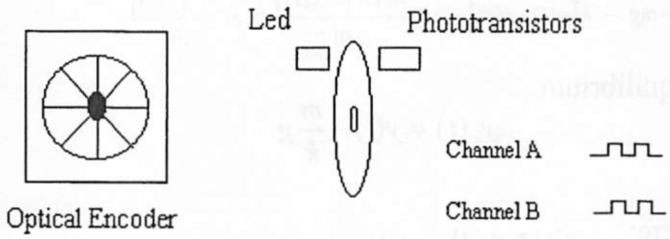


Proposed System

- Optical Encoder
  - To measure wheel rotation
- Slip of Wheels
  - Measure acceleration
  - Use Kalman Filter to obtain optimal estimate

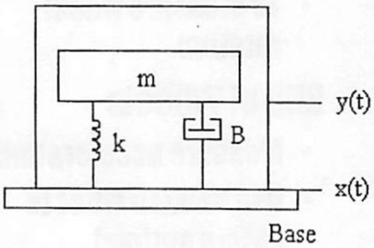


Optical Encoder



Accelerometer

- **Variables and parameters:**
  - **X(t) - Displacement of accelerometer base**
  - **y (t) - Displacement of seismic mass**
  - **m - Seismic mass**
  - **k - Spring constant**
  - **B - damping constant**



Accelerometer

Newton's Second Law:

$$-mg - k\left[y(t) - x(t)\right] - B\,\frac{\mathrm{d}\left[y(t)-x(t)\right]}{\mathrm{dt}} = m\,\frac{d^2\left[y(t)\right]}{dt^2}$$

in equilibrium

$$y_e(t) = y(t) - \frac{m}{k}g$$

Where: $z(t) = y_e(t) - x(t)$

$$m\,\frac{d^2\left[y(t)\right]}{dt^2} + B\frac{d\left[z(t)\right]}{dt} = -\,m\frac{d^2\left[x(t)\right]}{dt^2}$$

Accelerometer

Transfer function:

$$\frac{z(s)}{x(s)} = \frac{-ms^2}{ms^2 + Bs + k}$$

For low frequencies:

$$\omega << \omega_0 = \sqrt{(k/m)} \qquad \frac{z(s)}{x(s)} = \frac{-ms^2}{k}$$

Therefore, the relative position is proportional to the acceleration:

$$z(t) = \frac{-m}{k}\frac{d^2x(t)}{dt^2}$$

Kalman Filter

Stochastic System Model:

$$x_{k+1} = Ax_k + Gw_k$$
$$y_k = Cx_k + v_k$$
$$x_0 = \left(\overline{x_0}, P_0\right), w_k \approx (0, Q), v_k \approx (0, R)$$

Assumptions:

w_k and v_k are white noise processes mutually uncorrelated with each other and with x_0. $Q \geq 0,\ R > 0$

Filter Initialization:

$$\tilde{x}_0 = \overline{x_0}$$

Kalman Filter

Time Update:

Error Covariance: $P_{k+1} = AP_k A^T + GQG^T$

Estimate Update: $\tilde{x}_{k+1} = A\tilde{x}_k + Bu_k$

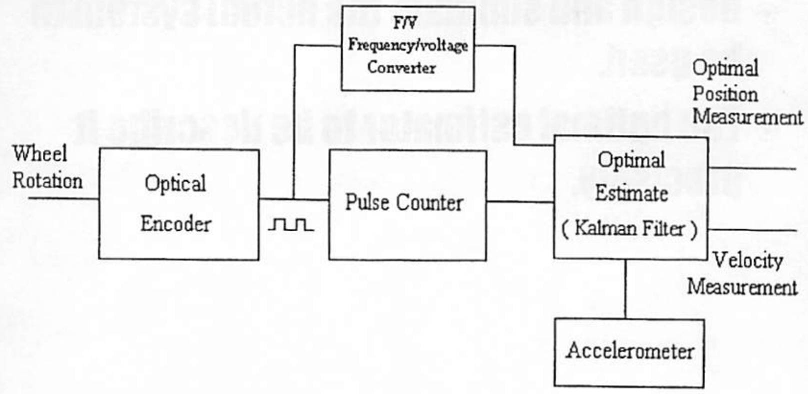
Measurement Update:

Error Covariance: $P_{k+1} = P^-_{k+1} - P^-_{k+1}C^T (C P^-_{k+1} C^T + R)^{-1} CP^-_{k+1}$

Estimate Update: $x_{k+1} = x^-_{k+1} + P_{k+1} C^T R^{-1} (y_{k+1} - Cx^-_{k+1})$

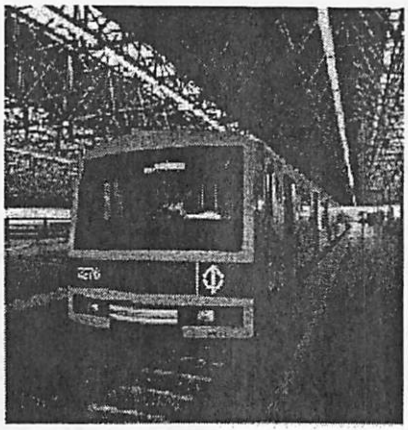
Kalman Gain: $K_{k+1} = P^-_{k+1}C^T (CP^-_{k+1} C^T + R)^{-1}$

Measuring System



Conclusions

- The proposed system
  - Provides the best estimate of train position given the measurements of wheel rotation and train acceleration.
  - Promises to be very reliable and robust



Further Research

- Design and simulate the actual system to be used.
- The optimal estimator to be describe it precisely.

Questions?

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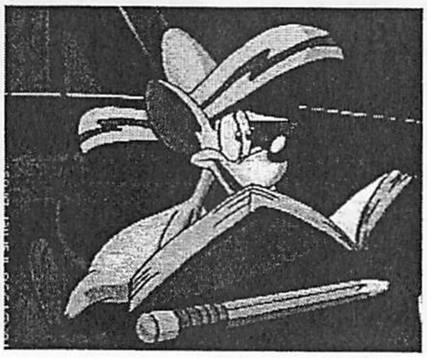

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